Day 200 **S Tier 10-a-day: Model Solutions**

1 HCF and LCM

(a) Write 686 and 1225 as products of prime factors.
(b) Find the HCF and the LCM of 686 and 1225.
Give your answers as numbers.

(a) $686 = 2 \times 7^3$ and $1225 = 5^2 \times 7^2$
(b) HCF $= 7^2 = 49$
LCM $= 2 \times 5^2 \times 7^3 = 17{,}150$
Answer: (a) 2×7^3; $5^2 \times 7^2$ (b) HCF = 49, LCM = 17,150

2 Inverse functions

$f(x) = \frac{-4x + 1}{3x + 1}$

(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(-1)$.

(a) Let $y = \frac{-4x + 1}{3x + 1}$. Then $y(3x + 1) = -4x + 1$
$3xy + y = -4x + 1$
Collect x terms: $3xy + 4x = 1 - y$
Factorise: $x(3y + 4) = -y + 1$, so $x = \frac{-y + 1}{3y + 4}$
$f^{-1}(x) = \frac{-x + 1}{3x + 4}$
(b) $f^{-1}(-1) = \frac{(-1) \times (-1) + 1}{3 \times (-1) - (-4)} = \frac{2}{1} = 2$
Answer: (a) $f^{-1}(x) = \frac{-x + 1}{3x + 4}$ (b) 2

3 Compound interest

Nia puts £2,500 in a savings account.
Compound interest is paid at 2% in year 1, 6% in year 2 and 1% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.02, 1.06, 1.01
£2,500 × 1.02 × 1.06 × 1.01
= £2,730.03
Answer: £2,730.03

4 Histograms with an unmarked scale

The histogram shows the ages of visitors to a museum.
280 of them had an age between 60 and 70 years.
(a) How many had an age between 40 and 60 years?
(b) How many visitors are there in total?

Frequency is proportional to the area of a bar
60 to 70: bar is 7 squares high and 10 wide, area 70, which represents 280
So 1 unit of area represents 280 ÷ 70 = 4
(a) 40 to 60: 6 × 20 = 120 units of area, so 120 × 4 = 480
(b) Total area = 80 + 60 + 120 + 70 = 330, so total = 330 × 4 = 1320
Answer: (a) 480 (b) 1320

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5 Similar shapes

Two shapes are mathematically similar.
The smaller has height 12 cm and area 20 cm^2.
The larger has area 45 cm^2.
Work out the height of the larger shape.

Ratio of areas = 45 : 20 = 9 : 4
Ratio of lengths = 3 : 2
Height = $12 \times \frac{3}{2} = 18$ cm

Answer: 18 cm

6 Trigonometric equations

Solve $2 \sin x° - 3 = -5$ for $0 \leq x \leq 360$

Rearrange: $2 \sin x° = -2$, so $\sin x° = -1$
From the graph of $y = \sin x°$, $\sin x° = -1$ when $x = 270$
Answer: x = 270

7 Line of best fit

The scatter graph shows age of car (x) and value (y).
The line of best fit passes through (2, 39) and (6, 37).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{37 - 39}{6 - 2} = \frac{-2}{4} = -0.5$
$y = -0.5x + c$: using (2, 39), $39 = -0.5 \times 2 + c$, so $c = 40$
$y = -0.5x + 40$
(b) For each extra year of age, the value of the car (in £100s) goes down by 0.5
Answer: (a) $y = -0.5x + 40$ (b) For each extra year of age, the value of the car (in £100s) goes down by 0.5

8 Reverse percentages

A teacher's salary is increased by 8% in one year.
The next year it is increased by 25%.
The teacher's salary is now £51,300 a year.
Work out the teacher's salary before the first change.

Multipliers: 1.08, 1.25
Overall multiplier = $1.08 \times 1.25 = 1.35$
Original = final ÷ multiplier = £51,300 ÷ 1.35
= £38,000
Answer: £38,000

9 Column vectors

The vectors $\begin{pmatrix} k \\ 24 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3k \end{pmatrix}$ are parallel.
k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same
$\frac{k}{2} = \frac{24}{3k}$
$k \times 3k = 24 \times 2$
$3k^2 = 48$
$k^2 = 16$
k = 4 (k is positive)
Answer: k = 4

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10 Finding the rate of interest

Hassan invested £11,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £12,438.40.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $11500 \times m^2 = 12438.40$

$m^2 = 12438.40 \div 11500 = 1.0816$

$m = \sqrt{1.0816} = 1.04$

Rate = $(1.04 - 1) \times 100 = 4\%$

Answer: 4% per year