Day 20 **S Tier 10-a-day: Model Solutions**

1 Rationalising the denominator

Write $\frac{10}{3+\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are integers.

Multiply top and bottom by $(3 - \sqrt{7})$:

$$\frac{10}{3+\sqrt{7}} = \frac{10(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$$

Denominator: $9 - 7 = 2$

$= 15 - 5\sqrt{7}$

Answer: $15 - 5\sqrt{7}$

2 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 12^2 + 3^2 = 153$
$AG^2 = AC^2 + CG^2 = 153 + 4^2 = 169$
AG = 13 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{153}}$

angle GAC = 17.9° (1 d.p.)
Answer: (a) 13 cm (b) 17.9°

3 Direct proportion formulae

E is directly proportional to the square root of v.
When v = 36, E = 36.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 12.

(a) $E = k\sqrt{v}$
$36 = k \times 6$, so $k = 36 \div 6 = 6$
$E = 6\sqrt{v}$

(b) $12 = 6\sqrt{v}$, so $\sqrt{v} = 12 \div 6 = 2$, giving $v = 4$

Answer: (a) $E = 6\sqrt{v}$ (b) $v = 4$

4 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = $(4x)°$
Angle ABC = $(2x - 18)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
$(4x) + 2(2x - 18) = 180$
$8x - 36 = 180$, so $8x = 216$
$x = 27$

(b) Angle BAC = $4 \times 27 + 0 = 108°$

Answer: (a) $x = 27$ (b) 108°

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5 Compound interest

Beth invests £1,000.
The account pays compound interest of 5% a year for the first 3 years,
then 2% a year for the next 2 years.
Work out the total amount after 5 years, to the nearest penny. (Calculator)

Multipliers: 1.05 for 3 years, then 1.02 for 2 years
£1,000 × 1.05^3 × 1.02^2
= £1,204.39
Answer: £1,204.39

6 Relative speed

Liam and Dina live 100 km apart.
At 10:00 they set off towards each other on the same road.
Liam drives at 40 km/h and Dina drives at 60 km/h.
(a) At what time do they meet?
(b) How far from Liam's home do they meet?

Closing speed = 40 + 60 = 100 km/h
(a) Time = 100 ÷ 100 = 1 hours = 60 minutes, so they meet at 11:00
(b) Distance = 40 × 1 = 40 km
Answer: (a) 11:00 (b) 40 km

7 Equations in disguise

Solve $x - 11\sqrt{x} + 30 = 0$

Let $u = \sqrt{x}$: $u^2 - 11u + 30 = 0$
Factorise: $(u - 6)(u - 5) = 0$, so $u = 6$ or $u = 5$
$\sqrt{x} = 6$ or $\sqrt{x} = 5$
$x = 36$ or $x = 25$
Answer: x = 25 or x = 36

8 Averages with algebra

Here are five numbers: $3x + 5$, $x + 2$, 6, $x - 1$, x.
The mean of the five numbers is 12.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(3x + 5) + (x + 2) + (6) + (x - 1) + (x) = 12 \times 5 = 60$
$6x + 12 = 60$, so $6x = 48$ and $x = 8$
The numbers are 29, 10, 6, 7, 8
(b) In order: 6, 7, 8, 10, 29, so the median is 8
Answer: (a) x = 8 (b) 8

9 Algebraic proof (harder)

Prove algebraically that $n(n + 1)(n + 2)(n + 3) + 1$ is a square number for all positive integer values of n.

$n(n + 3) = n^2 + 3n$ and $(n + 1)(n + 2) = n^2 + 3n + 2$
Let $m = n^2 + 3n$. Then the expression is $m(m + 2) + 1$
$= m^2 + 2m + 1 = (m + 1)^2$
$= (n^2 + 3n + 1)^2$, which is the square of an integer
Answer: Proved

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{5}\mathbf{a} = \frac{3}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$

Equate: $\frac{3}{5}s = 1 - t$ and $1 - s = \frac{1}{2}t$

Solving: $s = \frac{5}{7}$ and $t = \frac{4}{7}$

OX = $\frac{3}{7}\mathbf{a} + \frac{2}{7}\mathbf{b}$

Answer: (a) $\frac{3}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{7}\mathbf{a} + \frac{2}{7}\mathbf{b}$