Day 2 **S Tier 10-a-day: Model Solutions**

1 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 51°.
B is a point on the same line, 100 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 19°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 51° = \frac{h}{x}$, so $x = h \div \tan 51°$

From B: $\tan 19° = \frac{h}{x + 100}$, so $x + 100 = h \div \tan 19°$

Subtract: $100 = h \div \tan 19° - h \div \tan 51° = h(2.9042 - 0.80978)$

$h = 100 \div 2.0944 = 47.7$ m (1 d.p.)

Answer: 47.7 m

2 Fractions

Work out $(2\frac{4}{5} - 1\frac{1}{2}) \times \frac{1}{2}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{14}{5} - \frac{3}{2} = \frac{13}{10}$

$\frac{13}{10} \times \frac{1}{2} = \frac{13}{20}$

Answer: $\frac{13}{20}$

3 Histograms with an unmarked scale

The histogram shows the times of people spent waiting at a clinic.
70 of them had a time between 50 and 60 minutes.
(a) How many had a time between 30 and 50 minutes?
(b) How many people are there in total?

Frequency is proportional to the area of a bar
50 to 60: bar is 7 squares high and 10 wide, area 70, which represents 70
So 1 unit of area represents $70 \div 70 = 1$
(a) 30 to 50: $5 \times 20 = 100$ units of area, so $100 \times 1 = 100$
(b) Total area $= 10 + 120 + 100 + 70 = 300$, so total $= 300 \times 1 = 300$
Answer: (a) 100 (b) 300

4 Standard form

Work out $(7.5 \times 10^2) \div (5 \times 10^5)$

Give your answer in standard form.

$7.5 \div 5 = 1.5$

$10^2 \div 10^5 = 10^{-3}$

$= 1.5 \times 10^{-3}$

Answer: 1.5×10^{-3}

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5 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 31°} = \frac{16}{\sin 82°}$

$x = \frac{16 \times \sin 31°}{\sin 82°} = 8.3216$

x = 8.3 cm (1 d.p.)

Answer: AC = 8.3 cm

6 Equation of a circle

A and B are the points (−8, 1) and (−2, −1).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-8 + (-2)}{2}, \frac{1 + (-1)}{2}) = (-5, 0)$

$r^2 = (-8 - (-5))^2 + (1 - 0)^2 = 9 + 1 = 10$

$(x + 5)^2 + y^2 = 10$

Answer: $(x + 5)^2 + y^2 = 10$

7 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{20}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 5 days.
Give your answer correct to 3 significant figures. (Calculator)

$P(\text{never}) = (1 - \frac{1}{20})^5 = \frac{19}{20}^5 = 0.7737809375$

P(at least once) = 1 − P(never) = 1 − 0.7737809375
= 0.226

Answer: 0.226

8 Tangent to a circle

A circle has equation $(x - 3)^2 + (y - 3)^2 = 130$.
The point P(12, 10) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (3, 3). The gradient of the radius to P is $\frac{10 - 3}{12 - 3} = \frac{7}{9}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{9}{7}$. Using P: $y - 10 = -\frac{9}{7}(x - 12)$

Multiply by 7: $7(y - 10) = -9(x - 12)$
Expand and rearrange: $9x + 7y = 178$

Answer: 9x + 7y = 178

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9 Joint and inverse variation

y is directly proportional to the square of x and inversely proportional to the square of z.
When x = 2 and z = 2, y = 6.
Work out the value of y when x = 6 and z = 2.

$y = \frac{kx^2}{z^2}$

$6 = \frac{k \times 4}{4}$, so $k = 6 \times 4 \div 4 = 6$

$y = \frac{6 \times 36}{4} = 54$

Answer: y = 54

10 Tangency conditions

The line $y = kx - 6$ is a tangent to the curve $y = x^2 + 6$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 6 = kx - 6$

Rearrange: $x^2 - kx + 12 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 12 = 0$

$k^2 = 48$

$k = \pm\sqrt{48} = \pm 4\sqrt{3}$

Answer: $k = \pm 4\sqrt{3}$