Day 199 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 20:25 (London time).
It takes 9 hours 10 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 20:25 + 9 h 10 min = 05:35 the next day
Los Angeles is 8 hours behind London, so subtract 8 hours
21:35
Answer: 21:35 the same day

2 Enlargement

Triangle T has vertices A(−4, 6), B(8, 2), C(8, 0).

T is enlarged with scale factor $\frac{3}{2}$ and centre (4, 0).

(a) Write down the coordinates of the image of A.
(b) The area of T is 6 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (−4 − 4, 6 − 0) = (−8, 6)

Multiply by $\frac{3}{2}$: (−12, 9)

Image of A = centre + this vector = (−8, 9)

(b) Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$

Area = $6 \times \frac{9}{4} = \frac{27}{2}$ square units

Answer: (a) (−8, 9) (b) $\frac{27}{2}$

3 Probability and quadratic equations

A bag contains some counters. 7 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{21}{55}$.

Work out how many counters are white.

Let there be n counters altogether

P(both green) = $\frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)}$

$\frac{42}{n(n-1)} = \frac{21}{55}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

$(n - 11)(n + 10) = 0$, so $n = 11$ (n cannot be negative)

Number of counters that are white = 11 − 7 = 4

Answer: 4

4 Gradient of a chord

A and B are points on the curve $y = -x^2 - 6x + 5$.
The x coordinate of A is −3 and the x coordinate of B is 1.
Work out the gradient of the line AB.

At A: y = 14. At B: y = −2
A is (−3, 14) and B is (1, −2)

Gradient = $\frac{-2 - 14}{1 - (-3)} = \frac{-16}{4} = -4$

Answer: −4

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5 Percentages

Q is 25% more than C.
C is 30% less than B.
Work out Q as a percentage of B.

Q = 1.25 × C and C = 0.7 × B
Q = 1.25 × 0.7 × B = 0.875 × B
0.875 × 100 = 87.5%
Answer: 87.5%

6 Trigonometric equations

Solve $5 \tan x° + 2 = -3$ for $0 \leq x \leq 360$

Rearrange: $5 \tan x° = -5$, so $\tan x° = -1$
The acute angle with tangent 1 is 45°; tangent is negative in the second and fourth quadrants
x = 135 or x = 315 (180 − 45 and 360 − 45)
Answer: x = 135, 315

7 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 3) cm, (x + 5) cm and (2x + 4) cm. The longest side is (2x + 4) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 3)^2 + (x + 5)^2 = (2x + 4)^2$
Expand and collect terms: $2x^2 - 18 = 0$
Factorise: $2(x - 3)(x + 3) = 0$
x = 3 or x = −3, and x must be positive, so x = 3
(b) The two shorter sides are 6 cm and 8 cm
Area = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2
Answer: (a) x = 3 (b) 24 cm^2

8 Ratio and money transferred

Jamal and Noah share some money in the ratio 5 : 7.
Jamal gives Noah £23.
The ratio of their money is now 1 : 6.
How much money do they have altogether?

Let the amounts be 5k and 7k, so the total is 12k
Jamal now has 5k − 23, which is $\frac{1}{7}$ of the total
7(5k − 23) = 1 × 12k
35k − 161 = 12k, so 23k = 161 and k = 7
Total = 12 × 7 = £84
Answer: £84

9 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 4n - 9$.
(a) Work out the 8th term.
(b) Is 62 a term of the sequence? You must show your working.

(a) $2 \times 8^2 + 4 \times 8 - 9 = 151$
(b) Solve $2n^2 + 4n - 9 = 62$, so $2n^2 + 4n - 71 = 0$
$b^2 - 4ac = 4^2 - 4 \times 2 \times (-71) = 584$, which is not a square number
So n is not a whole number, and 62 is not a term
Answer: (a) 151 (b) No

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10 Recasting solids

A solid cone has base radius 6 cm and height 3 cm.
It is melted down and recast into a solid sphere.
Work out the radius of the sphere.

The volume of a cone is $\frac{1}{3}\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

Volume of cone = $\frac{1}{3} \times \pi \times 6^2 \times 3 = 36\pi$

$\frac{4}{3}\pi R^3 = 36\pi$, so $R^3 = 36 \times \frac{3}{4} = 27$

R = 3 cm

Answer: 3 cm