Day 198 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

60 beetles in a field are caught, marked and put back.
Later, a second sample of 120 beetles is caught. 10 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 10 out of 120

This should match the proportion marked in the whole population: $\frac{10}{120} = \frac{60}{N}$

N = 60 × 120 ÷ 10 = 720

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 720 (b) The population stays the same between samples

2 Similar shapes

Two shapes are mathematically similar.
The smaller has height 20 cm and area 36 cm^2.
The larger has area 5.76 cm^2.
Work out the height of the larger shape.

Ratio of areas = 5.76 : 36 = 4 : 25
Ratio of lengths = 2 : 5

Height = $20 \times \frac{2}{5}$ = 8 cm

Answer: 8 cm

3 Inequalities

(a) Solve $5 - 3x \geq 14$
(b) n is an integer. Write down all the values of n that satisfy
$-8 < 4n + 4 \leq 16$

(a) Subtract 5: $-3x \geq 9$
Divide by -3 and reverse the inequality sign: $x \leq -3$
(b) Subtract 4: $-12 < 4n \leq 12$
Divide by 4: $-3 < n \leq 3$
Integers: $-2, -1, 0, 1, 2, 3$
Answer: (a) $x \leq -3$ (b) $-2, -1, 0, 1, 2, 3$

4 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 3.
10% of the juniors and 80% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 80 students in total: 20 juniors and 60 seniors
10% of 20 = 2 and 80% of 60 = 48
Together: 50 out of 80
50 ÷ 80 × 100 = 62.5%
Answer: 62.5%

5 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 12) and (3, 108).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 12$ and $k\,a^3 = 108$

Divide: $a^2 = \frac{108}{12} = 9$

a = 3 (positive, since a > 0)
From the first point: k × 3 = 12, so k = 4
Answer: k = 4, a = 3

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6 Exact lengths and areas

A rectangle has area $(35 + 17\sqrt{5})$ cm^2 and length $(4 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{35 + 17\sqrt{5}}{4 + \sqrt{5}}$

Multiply top and bottom by $(4 - \sqrt{5})$:
Numerator: $(35 + 17\sqrt{5})(4 - \sqrt{5}) = 55 + 33\sqrt{5}$
Denominator: $16 - 5 = 11$
Width = $5 + 3\sqrt{5}$
Answer: $5 + 3\sqrt{5}$ cm

7 Radius from chord and height of arc

A circular arc has a chord AB of length 40 cm.
The greatest distance between the chord and the arc is 10 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 10$
AM = $40 \div 2 = 20$. Pythagoras in triangle OMA: $r^2 = (r - 10)^2 + 20^2$
$r^2 = r^2 - 20r + 100 + 400$
$20r = 500$, so $r = 25$ cm
Answer: 25 cm

8 Composite functions: fg and gf

$f(x) = 4x - 4$ and $g(x) = x^2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2) = 4x^2 - 4$
$gf(x) = g(4x - 4) = (4x - 4)^2 = 16x^2 - 32x + 16$
Set them equal and rearrange: $12x^2 - 32x + 20 = 0$
Divide by 4 and factorise: $(x - 1)(3x - 5) = 0$
$x = 1$ or $x = \frac{5}{3}$
Answer: $x = 1$ or $x = \frac{5}{3}$

9 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 60°.
B is a point on the same line, 80 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 40°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 60° = \frac{h}{x}$, so $x = h \div \tan 60°$

From B: $\tan 40° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 40°$
Subtract: $80 = h \div \tan 40° - h \div \tan 60° = h(1.1918 - 0.57735)$
$h = 80 \div 0.61440 = 130.2$ m (1 d.p.)
Answer: 130.2 m

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10 Bounds and accuracy

a = 0.567 and b = 0.977, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.5665 \leq a < 0.5675$; $0.9765 \leq b < 0.9775$
Lower bound = 0.5665 ÷ 0.9775 = 0.57954...
Upper bound = 0.5675 ÷ 0.9765 = 0.58116...
Both bounds round to 0.58 to 2 significant figures, but not to 3 significant figures
Answer: 0.58 (2 s.f.)