Day 197 **S Tier 10-a-day: Model Solutions**

1 Inverse proportion in context

9 workers can build a wall in 12 days.
After 4 days, 8 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 9 × 12 = 108 worker-days
Work done = 9 × 4 = 36, so work left = 72
Workers left = 9 − 8 = 1
Time = 72 ÷ 1 = 72 days
Answer: 72 days

2 Factorising quadratics

Factorise $6x^2 + 11x - 35$

6 × (−35) = −210. Two numbers that multiply to −210 and add to 11 are 21 and −10
Split the middle term: $6x^2 + 21x - 10x - 35$
Factorise in pairs: $3x(2x + 7) - 5(2x + 7)$
$= (3x - 5)(2x + 7)$
Answer: $(3x - 5)(2x + 7)$

3 Bounds in geometry

The radius of a circle is 7 cm, correct to the nearest centimetre.
Work out the lower bound of the area of the circle. Give your answer in terms of π.

The radius is between $\frac{13}{2}$ cm and $\frac{15}{2}$ cm

For the lower bound, use $r = \frac{13}{2}$

Area $= \pi \times (\frac{13}{2})^2 = \frac{169}{4}\pi$ cm^2

Answer: $\frac{169}{4}\pi$ cm^2

4 Surds and fractions

Simplify $\frac{2}{4 + \sqrt{10}} - \frac{1}{4 - \sqrt{10}}$
Give your answer with a rational denominator.

Common denominator: $(4 + \sqrt{10})(4 - \sqrt{10}) = 16 - 10 = 6$
Numerator: $2(4 - \sqrt{10}) - (4 + \sqrt{10})$
$= 4 - 3\sqrt{10}$
$\frac{4 - 3\sqrt{10}}{6}$
Answer: $\frac{4 - 3\sqrt{10}}{6}$

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
15, 29, 51, 81, 119
Find an expression for the nth term.

First differences: 14, 22, 30, 38
Second differences: 8, 8, 8, so $a = 8 \div 2 = 4$
Subtract $4n^2$ from each term: 11, 13, 15, 17, 19
The remaining linear sequence has nth term $2n + 9$
nth term $= 4n^2 + 2n + 9$
Answer: $4n^2 + 2n + 9$

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6 Finding a percentage rate

Ravi invests £6,500 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £8,651.50.
Work out the annual percentage interest rate.

Multiplier for 3 years = £8,651.50 ÷ £6,500 = 1.331
Yearly multiplier = the cube root of 1.331 = 1.1
10% increase each year
Answer: 10%

7 Conditional probability from a table

The table shows information about 122 employees.
(a) An employee who travels to work by bus is chosen at random. Work out the probability that this employee is a man.
(b) Two of the 122 employees are chosen at random. Work out the probability that both are men.

(a) 37 employees are in the Bus column, and 22 of them are men

Probability = $\frac{22}{37}$

(b) There are 64 men, so P(both) = $\frac{64}{122} \times \frac{63}{121} = \frac{2016}{7381}$

Answer: (a) $\frac{22}{37}$ (b) $\frac{2016}{7381}$

8 Equations in disguise

Solve $x^4 - 53x^2 + 196 = 0$

Let $u = x^2$: $u^2 - 53u + 196 = 0$
Factorise: $(u - 4)(u - 49) = 0$, so $u = 4$ or $u = 49$
$x^2 = 4$ or $x^2 = 49$
$x = \pm 2$ or $x = \pm 7$
Answer: $x = -7, -2, 2, 7$

9 Venn diagrams with algebra

The Venn diagram shows how many of 33 students study French (F) or Spanish (S).
(a) Work out the value of x.
(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: (x + 6) + x + (2x) + 3 = 33
4x + 9 = 33, so x = 6
(b) Students in the second circle: x + 2x = 6 + 12 = 18, of whom 6 are in both

Probability = $\frac{1}{3}$

Answer: (a) x = 6 (b) $\frac{1}{3}$

10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-2\mathbf{a} - 2\mathbf{b}$, OQ = $4\mathbf{a}$ and OR = $16\mathbf{a} + 4\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $6\mathbf{a} + 2\mathbf{b}$
QR = OR − OQ = $12\mathbf{a} + 4\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 4 = 1 : 2
Answer: (b) 1 : 2