**Day 196** **S Tier 10-a-day: Model Solutions**

## 1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x + 20)°
Angle ABC = (x + 5)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x + 20) + 2(x + 5) = 180
6x + 30 = 180, so 6x = 150
x = 25
(b) Angle BAC = 4 × 25 + 20 = 120°
**Answer: (a) x = 25 (b) 120°**

## 2 Fraction problems

A bag contains sweets.
$\frac{3}{10}$ of the sweets are orange and $\frac{1}{10}$ of the sweets are lemon.
The rest of the sweets, 12 of them, are strawberry.
How many orange sweets are there?

Fraction that are strawberry = $1 - \frac{3}{10} - \frac{1}{10} = \frac{3}{5}$

$\frac{3}{5}$ of the total is 12

Total = $12 \div \frac{3}{5} = 20$

Orange sweets = $\frac{3}{10}$ of 20 = 6

**Answer: 6**

## 3 Expanding three brackets

Expand and simplify $(x + 2)^2(x - 3)$

Expand the first two brackets: $(x + 2)^2 = x^2 + 4x + 4$
Multiply by $(x - 3)$: $x(x^2 + 4x + 4) - 3(x^2 + 4x + 4)$
$= x^3 + x^2 - 8x - 12$
**Answer: $x^3 + x^2 - 8x - 12$**

## 4 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 3 : 2.
Then 11 adults are removed and 12 children are removed.
The ratio is now 2 : 1.
How many children were there at the start?

Let the numbers at the start be 3k and 2k
(3k − 11) : (2k − 12) = 2 : 1, so 1(3k − 11) = 2(2k − 12)
3k − 11 = 4k − 24
−1k = −13, so k = 13
Number of children at the start: 2 × 13 = 26
**Answer: 26**

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## 5 Perpendicular bisector

A is the point (−3, 2) and B is the point (−11, 8).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−7, 5)

Gradient of AB = $-\frac{3}{4}$

Perpendicular gradient = $\frac{4}{3}$

$y - 5 = \frac{4}{3}(x - (-7))$

Rearrange to integer coefficients: $4x - 3y = -43$

**Answer: $4x - 3y = -43$**

## 6 Rationalising the denominator

Write $\frac{10}{2+\sqrt{5}}$ in the form $p + q\sqrt{5}$
where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{5})$:

$$\frac{10}{2+\sqrt{5}} = \frac{10(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$$

Denominator: $4 - 5 = -1$

$= -20 + 10\sqrt{5}$

**Answer: $-20 + 10\sqrt{5}$**

## 7 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 12, 12, 24, 72
Mid-points: 5, 20, 45, 80
$\Sigma fm = 12\times5 + 12\times20 + 24\times45 + 72\times80 = 7140$, $\Sigma f = 120$
Mean = 7140 ÷ 120 = 59.5
**Answer: 59.5**

## 8 Cone made from a sector

A sector of a circle has radius 30 cm and angle 288°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{288}{360} \times 2\pi \times 30 = 48\pi$ cm

This is the circumference of the base: $2\pi r = 48\pi$, so r = 24 cm

(b) Slant height = 30 cm, so height$^2 = 30^2 - 24^2 = 324$, height = 18 cm

Volume = $\frac{1}{3} \times \pi \times 24^2 \times 18 = 3456\pi$ cm$^3$

**Answer: (a) 24 cm (b) $3456\pi$ cm$^3$**

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## 9 Simultaneous equations

Solve the simultaneous equations
$-2x + 5y = -22$
$-5x + y = -32$

To match the x terms: (1) × 5 gives $-10x + 25y = -110$, and (2) × 2 gives $-10x + 2y = -64$
Subtract the equations to eliminate x: $23y = -46$
$y = -2$
Substitute $y = -2$ into (1): $-2x - 10 = -22$
$-2x = -12$, so $x = 6$
**Answer: $x = 6$, $y = -2$**

## 10 Finding the rate of interest

Beth invested £3,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £4,235.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $3500 \times m^2 = 4235.00$
$m^2 = 4235.00 \div 3500 = 1.21$
$m = \sqrt{1.21} = 1.1$
Rate = $(1.1 - 1) \times 100 = 10\%$
**Answer: 10% per year**