Day 195 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Rhys invests £8,000. The rates of interest for the next 3 years are 4%, 6% and 6%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.06 and 1.06
$8000 \times 1.04 \times 1.06 \times 1.06 = 9348.35$
To the nearest penny: £9,348.35
Answer: £9,348.35

2 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 20 ÷ 5 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 20 + 20 \times 8 + \frac{1}{2} \times 5 \times 20 = 50 + 160 + 50$
$= 260$
Answer: (a) 4 m/s^2 (b) 260 m

3 Pressure

A wooden block in the shape of a cuboid weighs 600 N.
Its smallest face has area 400 cm^2. Its largest face has area 800 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 400 cm^2 = 0.04 m^2 and 800 cm^2 = 0.08 m^2
(a) Greatest pressure is on the smallest face: 600 ÷ 0.04 = 15,000 N/m^2
(b) Least pressure is on the largest face: 600 ÷ 0.08 = 7,500 N/m^2
Answer: (a) 15,000 N/m^2 (b) 7,500 N/m^2

4 Fraction problems

A bag contains counters.
$\frac{3}{8}$ of the counters are red and $\frac{1}{3}$ of the counters are blue.
The rest of the counters, 21 of them, are green.
How many blue counters are there?

Fraction that are green = $1 - \frac{3}{8} - \frac{1}{3} = \frac{7}{24}$

$\frac{7}{24}$ of the total is 21

Total = $21 \div \frac{7}{24} = 72$

Blue counters = $\frac{1}{3}$ of 72 = 24

Answer: 24

5 Solving trigonometric equations

Solve $2 \sin x = 1$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\sin x = \frac{1}{2}$

The related acute angle satisfies $\sin \theta = \frac{1}{2}$, so $\theta = 30°$
sine is positive in the 1st and 2nd quadrants, so $x = 30°$ and $x = 180° - 30°$
$x = 30°$ and $x = 150°$
Answer: x = 30° or x = 150°

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6 Percentage change in area and volume

The side length of a square is decreased by 50%.
Work out the percentage change in the area of the square.

Side length multiplier = 0.5

Area multiplier = $0.5^2 = 0.25$

$0.25 - 1 = -0.75$, so a 75% decrease

Answer: 75% decrease

7 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{8-0}{0-(-3)} = \frac{8}{3}$

Perpendicular gradient = $-1 \div \frac{8}{3} = -\frac{3}{8}$

M passes through B(−3, 0): $0 = (-\frac{3}{8}) \times (-3) + c$, so $c = -\frac{9}{8}$

$y = -\frac{3}{8}x - \frac{9}{8}$

Answer: $y = -\frac{3}{8}x - \frac{9}{8}$

8 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 8^2 + 9^2 = 145$

$AG^2 = AC^2 + CG^2 = 145 + 12^2 = 289$

AG = 17 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{12}{\sqrt{145}}$

angle GAC = 44.9° (1 d.p.)

Answer: (a) 17 cm (b) 44.9°

9 Tangent to a circle: proof

Show that the line $y = x - 2$ is a tangent to the circle $x^2 + y^2 = 2$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x - 2$ into $x^2 + y^2 = 2$: $x^2 + (x-2)^2 = 2$

Expand: $2x^2 - 4x + 2 = 0$

$b^2 - 4ac = (-4)^2 - 4 \times 2 \times 2 = 16 - 16 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-4)}{2 \times 2} = 1$, and $y = 1 - 2 = -1$. The point is (1, −1)

Answer: (1, −1)

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10 Simple and compound interest

Ivan invests £12,000 for 4 years at 1.5% per year simple interest.
Priya invests £12,000 for 4 years at 1.5% per year compound interest.
Work out how much more interest Priya earns than Ivan.
Give your answer to the nearest penny. (Calculator)

Simple interest: £12,000 × 1.5% × 4 = £720

Compound: £12,000 × 1.015^4 = £12,736.36

Compound interest = £12,736.36 − £12,000 = £736.36

Difference = £736.36 − £720 = £16.36

Answer: £16.36