Day 194 **S Tier 10-a-day: Model Solutions**

1 Inequalities

(a) Solve $2 - 3x < 20$
(b) n is an integer. Write down all the values of n that satisfy
$-14 \leq 4n - 2 \leq 6$

(a) Subtract 2: $-3x < 18$
Divide by -3 and reverse the inequality sign: $x > -6$
(b) Add 2: $-12 \leq 4n \leq 8$
Divide by 4: $-3 \leq n \leq 2$
Integers: $-3, -2, -1, 0, 1, 2$
Answer: (a) $x > -6$ (b) $-3, -2, -1, 0, 1, 2$

2 Compound interest

Evan invests £7,000 at 3% per year compound interest.
After how many whole years will the investment first be worth more than £10,500?
(Calculator)

Multiplier = 1.03
After 13 years: $7000 \times 1.03^{13} = 10279.74$
After 14 years: $7000 \times 1.03^{14} = 10588.13$
10588.13 is the first value above £10,500
Answer: 14 years

3 Comparing costs

Zip Taxis has a fixed charge of £4 plus £3 per mile.
Go Cabs has a fixed charge of £22 plus £1.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 15 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $4 + 3n = 22 + 1.50n$
$1.50n = 18$, so $n = 12$
(b) Zip Taxis: £4 + £3 × 15 = £49
Go Cabs: £22 + £1.50 × 15 = £44.50
Go Cabs is cheaper by £4.50
Answer: (a) 12 (b) Go Cabs, by £4.50

4 Conditional probability from a table

The table shows information about 108 students.
(a) A student who studies French is chosen at random. Work out the probability that this student is a boy.
(b) Two of the 108 students are chosen at random. Work out the probability that both are boys.

(a) 21 students are in the French column, and 5 of them are boys

Probability = $\frac{5}{21}$

(b) There are 51 boys, so P(both) = $\frac{51}{108} \times \frac{50}{107} = \frac{425}{1926}$

Answer: (a) $\frac{5}{21}$ (b) $\frac{425}{1926}$

5 Profit and loss

A shop buys a guitar for £70.
It adds an 80% mark-up to get the selling price.
In a sale the selling price is reduced by 20%.
Work out the percentage profit or loss on the original cost.

Marked price = £70 × 1.8 = £126
Sale price = £126 × 0.8 = £100.80
Profit = £30.80

Percentage = $\frac{30.8}{70} \times 100 = 44\%$

Answer: 44% profit

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6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{3}{5}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{3}{5}\mathbf{a} = \frac{1}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) $\frac{1}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

7 Exact lengths and areas

A square has sides of length $(4 + \sqrt{6})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{6}$

Area = $(4 + \sqrt{6})^2$
= $16 + 8\sqrt{6} + 6$
= $22 + 8\sqrt{6}$

Answer: $22 + 8\sqrt{6}$ cm^2

8 Gradient of a chord

A and B are points on the curve $y = 3x^2 + 3x + 4$.
The x coordinate of A is −1 and the x coordinate of B is 0.
Work out the gradient of the line AB.

At A: y = 4. At B: y = 4
A is (−1, 4) and B is (0, 4)
Gradient = $\frac{4 - 4}{0 - (-1)} = \frac{0}{1} = 0$

Answer: 0

9 Nested surds

Show that $\sqrt{28 + 6\sqrt{3}} = 1 + 3\sqrt{3}$

Square the right-hand side: $(1 + 3\sqrt{3})^2 = 1 + 6\sqrt{3} + 27$
= $28 + 6\sqrt{3}$
So $\sqrt{28 + 6\sqrt{3}} = 1 + 3\sqrt{3}$

Answer: $\sqrt{28 + 6\sqrt{3}} = 1 + 3\sqrt{3}$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on OB with ON : NB = 3 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{5}\mathbf{a} = \frac{2}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{4}t\mathbf{b}$

Equate: $\frac{2}{5}s = 1 - t$ and $1 - s = \frac{3}{4}t$

Solving: $s = \frac{5}{14}$ and $t = \frac{6}{7}$

OX = $\frac{1}{7}\mathbf{a} + \frac{9}{14}\mathbf{b}$

Answer: (a) $\frac{2}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{7}\mathbf{a} + \frac{9}{14}\mathbf{b}$