Day 193 **S Tier 10-a-day: Model Solutions**

1 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on OB with ON : NB = 1 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{5}\mathbf{a} = \frac{2}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{3}t\mathbf{b}$

Equate: $\frac{2}{5}s = 1 - t$ and $1 - s = \frac{1}{3}t$

Solving: $s = \frac{10}{13}$ and $t = \frac{9}{13}$

OX = $\frac{4}{13}\mathbf{a} + \frac{3}{13}\mathbf{b}$

Answer: (a) $\frac{2}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{4}{13}\mathbf{a} + \frac{3}{13}\mathbf{b}$

2 Factorising fully

Factorise fully $5x^2 + 5x - 210$

Take out the common factor 5: $5(x^2 + x - 42)$
Factorise the quadratic: $5(x - 6)(x + 7)$
Answer: $5(x - 6)(x + 7)$

3 Simplifying surds

Write $\sqrt{180} - \sqrt{125} + \sqrt{5} + \sqrt{80}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{180} = 6\sqrt{5}$; $\sqrt{125} = 5\sqrt{5}$; $\sqrt{80} = 4\sqrt{5}$
$(6 - 5 + 1 + 4)\sqrt{5} = 6\sqrt{5}$
Answer: $6\sqrt{5}$

4 Exchange rates

Kira changes £400 into dollars at £1 = $1.15.
Kira spends $120 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Kira get back?

£400 × 1.15 = $460
460 − 120 = $340
340 ÷ 1.25 = £272
Answer: £272

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 30 ÷ 10 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 30 + 30 \times 12 + \frac{1}{2} \times 10 \times 30 = 150 + 360 + 150$
= 660
Answer: (a) 3 m/s^2 (b) 660 m

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6 Recasting solids

A solid cylinder of radius 10 cm and height 8 cm is melted down.
The metal is used to make identical solid spheres of radius 2 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 10^2 \times 8 = 800\pi$

Volume of one sphere = $\frac{4}{3} \times \pi \times 2^3 = \frac{32}{3}\pi$

Number of spheres = $800 \div \frac{32}{3} = 75$

Answer: 75

7 Growth and decay

A van is worth £2,000 when new.
Its value decreases by 8% each year.
After how many whole years will it first be worth less than £1,350? (Calculator)

Multiplier each year = 0.92

After 4 years: £2,000 × 0.92^4 = £1,432.79

After 5 years: £2,000 × 0.92^5 = £1,318.16

So the first time it is less than £1,350 is after 5 years

Answer: 5 years

8 Percentage change in area and volume

The side length of a square is decreased by 10%.
Work out the percentage change in the area of the square.

Side length multiplier = 0.9

Area multiplier = $0.9^2 = 0.81$

$0.81 - 1 = -0.19$, so a 19% decrease

Answer: 19% decrease

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(24, 7) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{7}{24}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{7}{24} = -\frac{24}{7}$

$y - 7 = (-\frac{24}{7})(x - 24)$

Multiply out and rearrange: 24x + 7y = 625

(b) On the x-axis y = 0, so 24x = 625

$x = \frac{625}{24}$, so T is $(\frac{625}{24}, 0)$

Answer: (a) 24x + 7y = 625 (b) T = $(\frac{625}{24}, 0)$

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10 Probability from a frequency tree

A factory makes 20,000 parts. 8% of them are faulty.
A scanner flags 75% of faulty parts and wrongly flags 5% of parts that are not faulty.
A part is chosen at random from the parts flagged by the scanner.
Work out the probability that this part is faulty.

8% of 20,000 = 1,600 are faulty and 18,400 are not
Flagged and genuine: 1,600 × 75% = 1,200
Flagged but not: 18,400 × 5% = 920
Total flagged or positive = 1,200 + 920 = 2,120

Probability = 1,200 ÷ 2,120 = $\frac{30}{53}$

Answer: $\mathbf{\frac{30}{53}}$