Day 192 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 34 is taken. Work out how many come from Group C.
(b) A different stratified sample has 30 from Group A. Work out the size of this sample.

Total = 200 + 200 + 200 + 80 = 680

(a) Group C: $\frac{200}{680} \times 34 = 10$

(b) Fraction sampled = $30 \div 200 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 680 = 102$

Answer: (a) 10 (b) 102

2 Forming and solving quadratic equations

A rectangle has length (3x + 3) cm and width (x + 1) cm.
The area of the rectangle is 192 cm^2.
(a) Show that $3x^2 + 6x - 189 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(3x + 3)(x + 1) = 192$, so $3x^2 + 6x + 3 = 192$
$3x^2 + 6x - 189 = 0$
(b) $3x^2 + 6x - 189 = 0$: $3(x - 7)(x + 9) = 0$
$x = 7$ ($x = -9$ is not possible as lengths are positive)
Length = 24 cm and width = 8 cm, so perimeter = $2 \times (24 + 8) = 64$ cm
Answer: (b) 64 cm

3 Error intervals

The mass of a parcel is 1400 kg, correct to the nearest 100.
Write down the error interval for the mass of a parcel, using m.

The value has been rounded to the nearest 100
Half of this is 50, so go 50 below and 50 above
1400 − 50 = 1350 and 1400 + 50 = 1450
$1350 \leq m < 1450$
Answer: $1350 \leq m < 1450$

4 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 80 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mark greater than 15.

(a) Median at cumulative frequency 40, so median = 40
Lower quartile at 20, so LQ = 30. Upper quartile at 60, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 15, cumulative frequency = 8
Number greater = 80 − 8 = 72
Answer: (a) median 40, IQR 20 (b) 72

5 Percentage change

The value of a car is decreased by 50%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 50% decrease it is 50.
To return to 100, increase by 50

Percentage increase = $\frac{50}{50} \times 100 = 100\%$

Answer: 100%

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6 Transformations of graphs

The graph of $y = f(x)$ has a minimum point at $(-4, 4)$.
Write down the coordinates of the turning point of the graph of
(a) $y = f(x + 4)$
(b) $y = f(x - 2) + 3$

(a) Start with the turning point $(-4, 4)$
Apply the transformation to the point: $(-8, 4)$
(b) $f(x - 2)$ changes the x-coordinate to -2, and $f(x) + 3$ changes the y-coordinate to 7
Turning point: $(-2, 7)$
Answer: (a) $(-8, 4)$ (b) $(-2, 7)$

7 Quadratic formula and surds

Solve $x^2 - 10x - 6 = 0$
Give your answers in the form $a \pm \sqrt{b}$

Complete the square: $(x - 5)^2 - 25 - 6 = 0$
$(x - 5)^2 = 31$
$x - 5 = \pm\sqrt{31}$
$x = 5 \pm \sqrt{31}$
Answer: $x = 5 \pm \sqrt{31}$

8 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 5 : 3.
Then 10 more apples are added and 2 pears are removed.
The ratio is now 11 : 5.
How many pears were there at the start?

Let the numbers at the start be $5k$ and $3k$
$(5k + 10) : (3k - 2) = 11 : 5$, so $5(5k + 10) = 11(3k - 2)$
$25k + 50 = 33k - 22$
$-8k = -72$, so $k = 9$
Number of pears at the start: $3 \times 9 = 27$
Answer: 27

9 Area and quadratic equations

The diagram shows a rectangle of length $(x + 5)$ cm, width $(x - 3)$ cm and area 84 cm^2.
(a) Show that $x^2 + 2x - 99 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(x + 5)(x - 3) = 84$
Expand: $x^2 + 2x - 15 = 84$, so $x^2 + 2x - 99 = 0$
(b) Factorise: $(x - 9)(x + 11) = 0$
$x = 9$ ($x = -11$ is not possible as a length would be negative)
Length = 14 cm and width = 6 cm
Perimeter $= 2 \times (14 + 6) = 40$ cm
Answer: (b) 40 cm

10 Standard form problems

A factory makes 1.3×10^4 bottles each day.
It runs for 2.5×10^2 days.
How many bottles does it make?
Give your answer in standard form.

Total $= (1.3 \times 10^4) \times (2.5 \times 10^2)$
$1.3 \times 2.5 = 3.25$ and $10^4 \times 10^2 = 10^6$
$= 3.25 \times 10^6$
Answer: 3.25×10^6