Day 191 **S Tier 10-a-day: Model Solutions**

1 Equations with algebraic fractions

Solve $\frac{4}{x-3} - \frac{6}{x+2} = \frac{1}{3}$

Multiply every term by $(x - 3)(x + 2)$: $4(x + 2) - 6(x - 3) = \frac{1}{3} \times (x - 3)(x + 2)$

Multiply by 3 to clear the fraction: $12(x + 2) - 18(x - 3) = (x - 3)(x + 2)$

Expand and rearrange so that one side is 0: $x^2 + 5x - 84 = 0$

Factorise: $(x - 7)(x + 12) = 0$

$x = -12$ or $x = 7$

Answer: $x = -12$ or $x = 7$

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 36°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle OAB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 36 = 72°$

(b) OA = OB (radii), so triangle OAB is isosceles

Angle OAB = $(180 - 72) \div 2 = 54°$

Answer: (a) 72° (b) 54°

3 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.

(a) Find an estimate for the median and the interquartile range.

(b) Estimate how many of the 80 had a mass greater than 45.

(a) Median at cumulative frequency 40, so median = 50

Lower quartile at 20, so LQ = 40. Upper quartile at 60, so UQ = 60

Interquartile range = $60 - 40 = 20$

(b) At 45, cumulative frequency = 30

Number greater = $80 - 30 = 50$

Answer: (a) median 50, IQR 20 (b) 50

4 Fraction problems

A bag contains marbles.

$\frac{1}{5}$ of the marbles are yellow and $\frac{2}{3}$ of the marbles are black.

The rest of the marbles, 6 of them, are white.

How many yellow marbles are there?

Fraction that are white = $1 - \frac{1}{5} - \frac{2}{3} = \frac{2}{15}$

$\frac{2}{15}$ of the total is 6

Total = $6 \div \frac{2}{15} = 45$

Yellow marbles = $\frac{1}{5}$ of 45 = 9

Answer: 9

5 Solving quadratics by factorising

Solve $4x^2 + 5 = 12x$

Rearrange so that one side is 0: $4x^2 - 12x + 5 = 0$

Factorise: $(2x - 1)(2x - 5) = 0$

$2x - 1 = 0$ or $2x - 5 = 0$

$x = \frac{1}{2}$ or $x = \frac{5}{2}$

Answer: $x = \frac{1}{2}$ or $x = \frac{5}{2}$

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6 Compound interest

Fatima invests £5,500 for 8 years at 6% per year compound interest.
Work out the value of the investment at the end of the 8 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 6 ÷ 100 = 1.06

$5500 \times 1.06^8 = 8766.16$

To the nearest penny: £8,766.16

Answer: £8,766.16

7 Volume and surface area of composite solids

A solid is made from a cylinder of height 6 cm and a hemisphere. Both have radius 3 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cylinder: $\pi \times 3^2 \times 6 = 54\pi$

Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 = 18\pi$

Total = $54\pi + 18\pi = 72\pi$ cm^3

Answer: 72π cm^3

8 Equation of a circle

A circle has centre C(−5, 1) and passes through the point P(1, −4).
Find an equation of the circle.

$r^2 = CP^2 = (1 - (-5))^2 + (-4 - 1)^2$

$= 36 + 25 = 61$

$(x + 5)^2 + (y - 1)^2 = 61$

Answer: $(x + 5)^2 + (y - 1)^2 = 61$

9 Surds with unknowns

Given that $(a + \sqrt{10})(b + \sqrt{10}) = 13 + 4\sqrt{10}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{10})(b + \sqrt{10}) = ab + (a + b)\sqrt{10} + 10$

Compare the $\sqrt{10}$ terms: a + b = 4

Compare the integer terms: ab + 10 = 13, so ab = 3

Two numbers with sum 4 and product 3 are 1 and 3

Answer: a = 1, b = 3

10 Tree diagrams and reverse probability

Machine A makes 50% of the bolts in a factory. Machine B makes the rest.
70% of the bolts from machine A are faulty.
20% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.5 × 0.7 + 0.5 × 0.2 = 0.35 + 0.1 = 0.45

(b) P(A and faulty) = 0.5 × 0.7 = 0.35

P(A | faulty) = 0.35 ÷ 0.45 = $\frac{7}{9}$

Answer: (a) 0.45 (b) $\frac{7}{9}$