Day 190 **S Tier 10-a-day: Model Solutions**

1 Column vectors

$\mathbf{a} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.

Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 5 - 1k \\ -1 + 1k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{5 - 1k}{1} = \frac{-1 + 1k}{5}$

$5(5 - 1k) = 1(-1 + 1k)$

$k \times (-6) = -26$

$k = \frac{13}{3}$

Answer: $\mathbf{k = \frac{13}{3}}$

2 Fractional and negative indices

Work out $27^{-\frac{2}{3}}$

A negative power means take the reciprocal: $27^{-\frac{2}{3}} = (\frac{1}{27})^{\frac{2}{3}}$

The denominator 3 means the cube root: the cube root of $\frac{1}{27}$ is $\frac{1}{3}$

The numerator 2 means raise to the power 2: $(\frac{1}{3})^2 = \frac{1}{9}$

Answer: $\frac{1}{9}$

3 Capture and recapture

60 beetles in a field are caught, marked and put back.

Later, a second sample of 40 beetles is caught. 5 of these are marked.

(a) Work out an estimate for the number of beetles in a field.

(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 40

This should match the proportion marked in the whole population: $\frac{5}{40} = \frac{60}{N}$

$N = 60 \times 40 \div 5 = 480$

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 480 (b) The population stays the same between samples

4 Rationalising the denominator

Rationalise the denominator of $\frac{4}{3\sqrt{5}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{5}$:

$\frac{4}{3\sqrt{5}} = \frac{4\sqrt{5}}{15}$

Answer: $\frac{4\sqrt{5}}{15}$

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5 Histograms

The histogram shows the lengths of leaves collected from a tree.
(a) Work out the total number of leaves.
(b) Estimate the number of leaves with a length between 20 cm and 45 cm.

Frequency = frequency density × class width
10 to 30: 0.8 × 20 = 16
30 to 40: 0.4 × 10 = 4
40 to 50: 0.4 × 10 = 4
50 to 80: 2 × 30 = 60
(a) Total = 84
(b) 20 to 30: 0.8 × 10 = 8
Whole bar between: 4
40 to 45: 0.4 × 5 = 2, so estimate = 14
Answer: (a) 84 (b) 14

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 60°
$\tan 60° = \sqrt{3}$
$x = 5 \times \tan 60° = 5 \times \sqrt{3}$
$x = 5\sqrt{3}$
Answer: $x = 5\sqrt{3}$

7 Forming and solving quadratic equations

The product of two positive numbers is 84.
The larger number is 5 more than the smaller number.
Let the smaller number be n.
(a) Show that $n^2 + 5n - 84 = 0$
(b) Work out the two numbers.

(a) The numbers are n and n + 5, so $n(n + 5) = 84$
$n^2 + 5n = 84$, so $n^2 + 5n - 84 = 0$
(b) Factorise: $(n - 7)(n + 12) = 0$
$n = 7$ or $n = -12$. n is positive, so $n = 7$
The numbers are 7 and 12
Answer: (b) 7 and 12

8 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are x cm, (x − 1) cm and (x + 1) cm.
The longest side is (x + 1) cm.
(a) Show that $x^2 - 4x = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $x^2 + (x - 1)^2 = (x + 1)^2$
Expand: $x^2 + x^2 - 2x + 1 = x^2 + 2x + 1$
Rearrange: $x^2 - 4x = 0$
(b) Factorise: $(x - 4)x = 0$, so $x = 4$ (the other root, $x = 0$, does not give valid side lengths)
The shorter sides are 4 cm and 3 cm, so area = $\frac{1}{2} \times 4 \times 3 = 6$ cm^2
Answer: (b) 6 cm^2

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9 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 62° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{11} = \frac{\sin 62°}{18}$

$\sin\theta = \frac{11 \times \sin 62°}{18} = 0.53958$

$\theta = \sin^{-1}(0.53958) = 32.7°$ (1 d.p.)

Answer: θ = 32.7°

10 Compound interest

Uma invests £9,500 for 5 years at 4% per year compound interest.
Work out the value of the investment at the end of the 5 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 4 ÷ 100 = 1.04

$9500 \times 1.04^5 = 11558.20$

To the nearest penny: £11,558.20

Answer: £11,558.20