Day 19 **S Tier 10-a-day: Model Solutions**

1 Simplifying surds

Write $\sqrt{252} + \sqrt{28} - \sqrt{175} - \sqrt{7}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{252} = 6\sqrt{7}$; $\sqrt{28} = 2\sqrt{7}$; $\sqrt{175} = 5\sqrt{7}$
$(6 + 2 - 5 - 1)\sqrt{7} = 2\sqrt{7}$
Answer: $2\sqrt{7}$

2 Solving trigonometric equations

Solve $2 \cos x = 1$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\cos x = \frac{1}{2}$

The related acute angle satisfies $\cos \theta = \frac{1}{2}$, so $\theta = 60°$

cosine is positive in the 1st and 4th quadrants, so $x = 60°$ and $x = 360° - 60°$
$x = 60°$ and $x = 300°$
Answer: $x = 60°$ or $x = 300°$

3 Quadratic inequalities

Solve $2x^2 < 8x$

Rearrange so that one side is 0: $2x^2 - 8x < 0$
Divide by 2: $x^2 - 4x < 0$
Solve $x^2 - 4x = 0$: $(x)(x - 4) = 0$, so $x = 0$ or $x = 4$
The graph is a U shape crossing the x-axis at 0 and 4; it is below the axis between the roots
$0 < x < 4$
Answer: $0 < x < 4$

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 54°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 54 = 108°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 108 = 72°$
Answer: (a) 108° (b) 72°

5 Fractions removed from a ratio

The ratio of men to women people at a meeting is 3 : 5.
Then $\frac{1}{3}$ of the men and $\frac{1}{5}$ of the women are taken out.
Work out the ratio of men to women now.

Left: $\frac{2}{3} \times 3 = 2$ of the men part, $\frac{4}{5} \times 5 = 4$ of the women part
2 : 4
Multiply by 1: 2 : 4 = 1 : 2
Answer: 1 : 2

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6 Reverse percentages

The value of a house is decreased by 12% in one year.
The next year its value is decreased by 30%.
It is now worth £135,520.
Work out the value of the house before the first change.

Multipliers: 0.88, 0.7
Overall multiplier = 0.88 × 0.7 = 0.616
Original = final ÷ multiplier = £135,520 ÷ 0.616
= £220,000
Answer: £220,000

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 1) cm, (x + 2) cm and (x + 3) cm.
The longest side is (x + 3) cm.
(a) Show that $x^2 - 4 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 1)^2 + (x + 2)^2 = (x + 3)^2$
Expand: $x^2 + 2x + 1 + x^2 + 4x + 4 = x^2 + 6x + 9$
Rearrange: $x^2 - 4 = 0$
(b) Factorise: $(x - 2)(x + 2) = 0$, so $x = 2$ (the other root, $x = -2$, does not give valid side lengths)
The shorter sides are 3 cm and 4 cm, so area = $\frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (b) 6 cm^2

8 Ratio and expressions

x : y = 9 : 4
Work out the ratio (1x − 1y) : (2x + 4y)
Give your answer in its simplest form.

x : y = 9 : 4, so let x = 9t and y = 4t
1(9t) − 1(4t) = 5t
2(9t) + 4(4t) = 34t
5t : 34t = 5 : 34
Answer: 5 : 34

9 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = 2x^2 + x - 1$.
Find the value of k.

Equate: $2x^2 + x - 1 = -2x + k$
Rearrange: $2x^2 + 3x - 1 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $9 - 4 \times 2 \times (-1 - k) = 0$
$8k + 17 = 0$
$k = -\frac{17}{8}$

Answer: $k = -\frac{17}{8}$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on OB with ON : NB = 3 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{5}\mathbf{a} = \frac{2}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{5}t\mathbf{b}$

Equate: $\frac{2}{5}s = 1 - t$ and $1 - s = \frac{3}{5}t$

Solving: s = $\frac{10}{19}$ and t = $\frac{15}{19}$

OX = $\frac{4}{19}\mathbf{a} + \frac{9}{19}\mathbf{b}$

Answer: (a) $\frac{2}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{4}{19}\mathbf{a} + \frac{9}{19}\mathbf{b}$