Day 189 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3p - q^3$
when $p = 3$, $q = 4$ and $r = -1$.

Substitute the values: $3 \times 3 - 4^3$
$= 9 - 64$
$= -55$
Answer: -55

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 46°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 46 = 92°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 92 = 88°$
Answer: (a) 92° (b) 88°

3 Averages with algebra

Here are five numbers: 2, x, 3x, 3x + 3, x − 2.
The mean of the five numbers is 15.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (2) + (x) + (3x) + (3x + 3) + (x − 2) = $15 \times 5 = 75$
$8x + 3 = 75$, so $8x = 72$ and $x = 9$
The numbers are 2, 9, 27, 30, 7
(b) Range = $30 - 2 = 28$
Answer: (a) x = 9 (b) 28

4 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 28°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 28°
Angle COB is an exterior angle of triangle OAC: 28 + 28 = 56°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 56°

5 Prime factors and indices

Write $10^4 \times 12^4$ as a product of powers of prime numbers.

$10 = 2 \times 5$, $12 = 2^2 \times 3$
$10^4 = 2^4 \times 5^4$
$12^4 = 2^8 \times 3^4$
Add the powers of each prime: $2^{12} \times 3^4 \times 5^4$
Answer: $2^{12} \times 3^4 \times 5^4$

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6 Line and curve intersections

Solve the simultaneous equations

$y = x^2 + 3x - 2$

$y = 2x$

Substitute: $x^2 + 3x - 2 = 2x$

Rearrange: $x^2 + x - 2 = 0$

Factorise: $(x - 1)(x + 2) = 0$, so $x = 1$ or $x = -2$

When $x = 1$: $y = 2 \times 1 = 2$

When $x = -2$: $y = 2 \times (-2) = -4$

Answer: $x = -2$, $y = -4$ and $x = 1$, $y = 2$

7 Density of an alloy

An alloy is made by mixing 60 cm^3 of lead with 20 cm^3 of iron.

The density of lead is 11.3 g/cm^3. The density of iron is 7.9 g/cm^3.

Work out the density of the alloy.

Mass of lead = $60 \times 11.3 = 678$ g

Mass of iron = $20 \times 7.9 = 158$ g

Total mass = 836 g, total volume = 80 cm^3

Density = $836 \div 80 = 10.45$ g/cm^3

Answer: 10.45 g/cm^3

8 Iteration: rearranging equations

(a) Show that the equation $x^2 - 6x - 5 = 0$ can be rearranged to give $x = 6 + \frac{5}{x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = 6 + \frac{5}{x_n}$ with $x_0 = 6$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^2 - 6x - 5 = 0$: $x^2 - 6x = 5$

Factorise: $x(x - 6) = 5$

Divide by x: $x - 6 = \frac{5}{x}$, so $x = 6 + \frac{5}{x}$

(b) $x_1 = 6 + \frac{5}{6} = 6.83333...$

$x_2 = 6 + \frac{5}{x_1} = 6.73171...$

$x_1 = 6.833$ and $x_2 = 6.732$ (3 d.p.)

Answer: (b) $x_1 = 6.833$, $x_2 = 6.732$

9 Laws of indices

Write $\frac{9^n \times 3^{n-2}}{9^n}$ in the form 3^{an+c}

where a and c are integers.

$9 = 3^2$, $3 = 3^1$, $9 = 3^2$

$9^n = 3^{2(n)}$, $3^{n-2} = 3^{n-2}$, $9^n = 3^{2n}$

Exponent: $2(n) + (n - 2) - 2n = n - 2$

Answer: $a = 1$, $c = -2$

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10 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 59°.
B is a point on the same line, 50 m further from the mast than A.
The angle of elevation of the top of the mast from B is 49°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 59° = \frac{h}{x}$, so $x = h \div \tan 59°$

From B: $\tan 49° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 49°$

Subtract: $50 = h \div \tan 49° - h \div \tan 59° = h(0.86929 - 0.60086)$
$h = 50 \div 0.26843 = 186.3$ m (1 d.p.)
Answer: 186.3 m