Day 188 **S Tier 10-a-day: Model Solutions**

1 Inequalities

(a) Solve $-3 - 6x \leq -3$
(b) n is an integer. Write down all the values of n that satisfy
$7 \leq 3n + 4 < 25$

(a) Add 3: $-6x \leq 0$
Divide by -6 and reverse the inequality sign: $x \geq 0$
(b) Subtract 4: $3 \leq 3n < 21$
Divide by 3: $1 \leq n < 7$
Integers: 1, 2, 3, 4, 5, 6
Answer: (a) $x \geq 0$ (b) 1, 2, 3, 4, 5, 6

2 Squares, cubes and prime factors

Find the smallest positive integer k such that $4500 \times k$ is a cube number.

$4500 = 2^2 \times 3^2 \times 5^3$
For a cube number every power must be a multiple of 3
Multiply by 2×3 to get $2^3 \times 3^3 \times 5^3$
k = 6
Answer: k = 6

3 Exchange rates

Callum changes £1200 into dollars at £1 = $1.2.
Callum spends $1240 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Callum get back?

£1200 × 1.2 = $1440
1440 − 1240 = $200
200 ÷ 1.25 = £160
Answer: £160

4 Percentages

D is 12% more than P.
P is 40% less than B.
Work out D as a percentage of B.

D = 1.12 × P and P = 0.6 × B
D = 1.12 × 0.6 × B = 0.672 × B
0.672 × 100 = 67.2%
Answer: 67.2%

5 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 45°

$\cos 45° = \frac{\sqrt{2}}{2}$

$x = 6 \times \cos 45° = 6 \times \frac{\sqrt{2}}{2}$

$x = 3\sqrt{2}$

Answer: $x = 3\sqrt{2}$

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6 Tangent to a circle

A circle has equation $(x + 3)^2 + (y - 3)^2 = 130$.
The point P(8, 6) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is $(-3, 3)$. The gradient of the radius to P is $\frac{6 - 3}{8 - (-3)} = \frac{3}{11}$

The tangent is perpendicular to the radius, so its gradient is $-\frac{11}{3}$. Using P: $y - 6 = -\frac{11}{3}(x - 8)$

Multiply by 3: $3(y - 6) = -11(x - 8)$
Expand and rearrange: $11x + 3y = 106$
Answer: 11x + 3y = 106

7 Transformations of graphs

The graph of y = f(x) has a minimum point at $(-2, 2)$.
Write down the coordinates of the turning point of the graph of
(a) y = f(x + 4)
(b) y = 3f(x − 4)

(a) Start with the turning point $(-2, 2)$
Apply the transformation to the point: $(-6, 2)$
(b) f(x − 4) changes the x-coordinate to 2, and 3f(x) changes the y-coordinate to 6
Turning point: (2, 6)
Answer: (a) (−6, 2) (b) (2, 6)

8 Equations with algebraic fractions

Solve $\frac{1}{x - 2} - \frac{6}{x + 2} = \frac{2}{3}$

Multiply every term by $(x - 2)(x + 2)$: $(x + 2) - 6(x - 2) = \frac{2}{3} \times (x - 2)(x + 2)$
Multiply by 3 to clear the fraction: $3(x + 2) - 18(x - 2) = 2(x - 2)(x + 2)$
Expand and rearrange so that one side is 0: $2x^2 + 15x - 50 = 0$
Factorise: $(x + 10)(2x - 5) = 0$
$x = -10$ or $x = \frac{5}{2}$

Answer: $x = -10$ or $x = \frac{5}{2}$

9 Combining means

The mean of the marks in a test for the Year 10 students is 42.
The mean of the marks in a test for the Year 11 students is 63.
The mean for all of the Year 10 students and Year 11 students together is 54.25.
Work out the ratio of the number of Year 10 students to the number of Year 11 students.

Let there be p Year 10 students and q Year 11 students
Total = 42p + 63q, and the overall mean gives $42p + 63q = 54.25(p + q)$
$-12.25p = -8.75q$
$p : q = -8.75 : -12.25 = 5 : 7$
Answer: 5 : 7

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10 Surds and fractions

Simplify $\frac{2}{2+\sqrt{2}} + \frac{1}{2-\sqrt{2}}$

Give your answer with a rational denominator.

Common denominator: $(2+\sqrt{2})(2-\sqrt{2}) = 4 - 2 = 2$

Numerator: $2(2-\sqrt{2}) + (2+\sqrt{2})$

$= 6 - \sqrt{2}$

$\frac{6-\sqrt{2}}{2}$

Answer: $\mathbf{\frac{6-\sqrt{2}}{2}}$