Day 187 **S Tier 10-a-day: Model Solutions**

1 Pressure

A stone block in the shape of a cuboid weighs 270 N.
Its smallest face has area 250 cm^2. Its largest face has area 400 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 250 cm^2 = 0.025 m^2 and 400 cm^2 = 0.04 m^2
(a) Greatest pressure is on the smallest face: 270 ÷ 0.025 = 10,800 N/m^2
(b) Least pressure is on the largest face: 270 ÷ 0.04 = 6,750 N/m^2
Answer: (a) 10,800 N/m^2 (b) 6,750 N/m^2

2 Tests of divisibility

The 4-digit number 342k is a multiple of 12.
k is a single digit.
Find all the possible values of k.

A multiple of 12 is a multiple of 3 and a multiple of 4
Digit sum = 3 + 4 + 2 + k = 9 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Last two digits 2k must be a multiple of 4: k = 0
Both conditions give k = 0
Answer: k = 0

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 35°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 35°
Angle COB is an exterior angle of triangle OAC: 35 + 35 = 70°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 70°

4 Quadratic inequalities

Solve $2x^2 - 10x > 0$

Divide by 2: $x^2 - 5x > 0$
Solve $x^2 - 5x = 0$: $(x)(x - 5) = 0$, so x = 0 or x = 5
The graph is a U shape crossing the x-axis at 0 and 5; it is above the axis outside the roots
$x < 0$ or $x > 5$
Answer: $x < 0$ or $x > 5$

5 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the product is greater than 8.
(b) Given that the first number is greater than the second, work out the probability that the product is greater than 8.

(a) There are 4 × 6 = 24 equally likely outcomes, and 10 of them satisfy: the product is greater than 8
Probability = $\frac{5}{12}$
(b) 6 outcomes satisfy: the first number is greater than the second. Of these, 1 also satisfy: the product is greater than 8
Probability = $\frac{1}{6}$
Answer: (a) $\frac{5}{12}$ (b) $\frac{1}{6}$

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6 Algebraic fractions: adding and subtracting

Write $\frac{2}{x+2} - \frac{x-4}{x-2}$ as a single fraction.

Simplify the numerator.

The common denominator is $(x + 2)(x - 2)$

$\frac{2}{x+2} - \frac{x-4}{x-2} = \frac{2(x-2) - (x-4)(x+2)}{(x+2)(x-2)}$

Expand the numerator: $-x^2 + 4x + 4$

$\frac{-x^2 + 4x + 4}{(x+2)(x-2)}$

Answer: $\mathbf{\frac{-x^2 + 4x + 4}{(x+2)(x-2)}}$

7 Squares, cubes and prime factors

Find the smallest positive integer k such that $18 \times k$ is a cube number.

$18 = 2 \times 3^2$

For a cube number every power must be a multiple of 3

Multiply by $2^2 \times 3$ to get $2^3 \times 3^3$

$k = 12$

Answer: k = 12

8 Perpendicular bisector

A is the point $(-3, 5)$ and B is the point $(-7, -3)$.

Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(-5, 1)$

Gradient of AB = 2

Perpendicular gradient = $-\frac{1}{2}$

$y - 1 = (-\frac{1}{2})(x - (-5))$

Rearrange to integer coefficients: $x + 2y = -3$

Answer: x + 2y = −3

9 Laws of indices

Write $\frac{3^n \times 9^{n-2}}{9^n}$ in the form 3^{an+c}

where a and c are integers.

$3 = 3^1$, $9 = 3^2$, $9 = 3^2$

$3^n = 3^n$, $9^{n-2} = 3^{2(n-2)}$, $9^n = 3^{2n}$

Exponent: $(n) + 2(n - 2) - 2n = n - 4$

Answer: a = 1, c = −4

10 Tangency conditions

The line $y = kx - 5$ is a tangent to the curve $y = x^2 + 2$.

Find the possible values of k.

Give any surds in their simplest form.

Equate: $x^2 + 2 = kx - 5$

Rearrange: $x^2 - kx + 7 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 7 = 0$

$k^2 = 28$

$k = \pm\sqrt{28} = \pm 2\sqrt{7}$

Answer: $\mathbf{k = \pm 2\sqrt{7}}$