Day 186 **S Tier 10-a-day: Model Solutions**

1 Rational and irrational numbers

Here are six numbers.

$\sqrt{50}$, $\pi \div 2$, $\sqrt{3} \times \sqrt{5}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $(1+\sqrt{2})^2$, $\sqrt[3]{27}$

Write down the rational numbers.

Simplify each number:

$\sqrt{50} = 5\sqrt{2}$; $\pi \div 2$ is irrational; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $(1+\sqrt{2})^2 = 3 + 2\sqrt{2}$; $\sqrt[3]{27} = 3$

Answer: $\frac{\sqrt{18}}{\sqrt{2}}$, $\sqrt[3]{27}$

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 40 ÷ 2 = 20 cm

$OA^2 = OM^2 + AM^2 = 21^2 + 20^2 = 841$

OA = 29 cm

(b) sin(angle AOM) = $\frac{20}{29}$

Angle AOM = 43.60°, so angle AOB = 2 × 43.60 = 87.2°

Answer: (a) 29 cm (b) 87.2°

3 Rationalising the denominator

Write $\frac{15}{2-\sqrt{5}}$ in the form $p + q\sqrt{5}$
where p and q are integers.

Multiply top and bottom by $(2+\sqrt{5})$:

$\frac{15}{2-\sqrt{5}} = \frac{15(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$

Denominator: $4 - 5 = -1$

$= -30 - 15\sqrt{5}$

Answer: $-30 - 15\sqrt{5}$

4 Composite functions

$f(x) = 2x + 5$ and $g(x) = \frac{8}{x}$

(a) Find gf(x).
(b) Solve gf(x) = 8

(a) $gf(x) = g(2x+5) = \frac{8}{2x+5}$

(b) $\frac{8}{2x+5} = 8$, so $2x + 5 = 8 \div 8 = 1$

$2x = -4$

$x = -2$

Answer: (a) $\frac{8}{2x+5}$ (b) $x = -2$

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5 Three-part ratios

Hassan, Kira and Jamal share some money.
Hassan : Kira = 4 : 7
Kira : Jamal = 8 : 3
(a) Work out the ratio Hassan : Kira : Jamal.
(b) They share £545 altogether. How much money does Jamal get?

(a) Make the Kira part the same: 4 : 7 = 32 : 56 and 8 : 3 = 56 : 21
Hassan : Kira : Jamal = 32 : 56 : 21
(b) Total parts = 109, so 1 part = 545 ÷ 109 = 5
Jamal gets 21 × 5 = £105
Answer: (a) 32 : 56 : 21 (b) £105

6 Combined transformations

The point P(4, 1) is rotated 90° anticlockwise about the origin.
The image is then reflected in the line $y = -x$.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−1, 4)
After the second transformation: P'' = (−4, 1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−y, x) → (−x, y)
The single mapping is (x, y) → (−x, y)
This is a reflection in the y-axis
Answer: (a) (−4, 1) (b) Reflection in the y-axis

7 At least one

The probability that a light bulb in a hallway fails in any month is $\frac{3}{10}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 5 months.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{3}{10})^5 = \frac{7}{10}^5 = 0.16807$
P(at least once) = 1 − P(never) = 1 − 0.16807
= 0.832
Answer: 0.832

8 Proof with indices

Show that $3^{m+2} + 3^m$ is always a multiple of 10.

$3^{m+2} = 3^m \times 3^2$
$3^{m+2} + 3^m = 3^m(3^2 + 1)$
$3^2 + 1 = 10$
$= 10 \times 3^m$, which is a multiple of 10
Answer: 10×3^m

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9 Functions that are their own inverse

$f(x) = \frac{-x-6}{3x+1}$ where $x \neq -\frac{1}{3}$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (3x + 1)

$ff(x) = \frac{(-1)(-x-6)+(-6)(3x+1)}{3(-x-6)-(-1)(3x+1)}$

Top = $x + 6 - 18x - 6 = -17x$

Bottom = $-3x - 18 + 3x + 1 = -17$

$ff(x) = \frac{-17x}{-17} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-x-6}{3x+1}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{-x-6}{3x+1}}$

10 Similar shapes

Two solid shapes, A and B, are mathematically similar.

The surface area of A is 12 cm^2 and the surface area of B is 48 cm^2.

The volume of A is 125 cm^3.

Work out the volume of B.

Ratio of areas = 12 : 48 = 1 : 4

Ratio of lengths = $\sqrt{1} : \sqrt{4}$ = 1 : 2

Ratio of volumes = $1^3 : 2^3$, so scale factor = 8

Volume of B = 125 × 8 = 1000 cm^3

Answer: 1000 $\mathbf{cm^3}$