Day 185 **S Tier 10-a-day: Model Solutions**

1 Factorising fully

Factorise fully $3x^2 - 3x - 90$

Take out the common factor 3: $3(x^2 - x - 30)$
Factorise the quadratic: $3(x + 5)(x - 6)$
Answer: $3(x + 5)(x - 6)$

2 Exchange rates

Leila changes £900 into dollars at £1 = $1.18.
Leila spends $120 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Leila get back?

£900 × 1.18 = $1062
1062 − 120 = $942
942 ÷ 1.25 = £753.60
Answer: £753.60

3 Similar shapes

Two solid shapes, A and B, are mathematically similar.
The surface area of A is 18 cm^2 and the surface area of B is 162 cm^2.
The volume of A is 100 cm^3.
Work out the volume of B.

Ratio of areas = 18 : 162 = 1 : 9
Ratio of lengths = $\sqrt{1} : \sqrt{9}$ = 1 : 3
Ratio of volumes = $1^3 : 3^3$, so scale factor = 27
Volume of B = 100 × 27 = 2700 cm^3
Answer: 2700 cm^3

4 Venn diagrams with algebra

The Venn diagram shows how many of 29 students play the piano (P) or the guitar (G).
(a) Work out the value of x.
(b) A student is chosen at random from those who play the guitar. Work out the probability that this student also plays the piano.

(a) Total: $(x + 5) + x + (2x - 3) + 7 = 29$
$4x + 9 = 29$, so $x = 5$
(b) Students in the second circle: $x + 2x - 3 = 5 + 7 = 12$, of whom 5 are in both
Probability = $\frac{5}{12}$
Answer: (a) x = 5 (b) $\frac{5}{12}$

5 Rationalising the denominator

Write $\frac{13}{2 - \sqrt{6}}$ in the form $\frac{p + q\sqrt{6}}{r}$
where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(2 + \sqrt{6})$:
$\frac{13}{2 - \sqrt{6}} = \frac{13(2 + \sqrt{6})}{(2 - \sqrt{6})(2 + \sqrt{6})}$
Denominator: $4 - 6 = -2$
$= \frac{-26 - 13\sqrt{6}}{2}$
Answer: $\frac{-26 - 13\sqrt{6}}{2}$

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6 Direct and inverse proportion

F is inversely proportional to the square root of d.
When d = 4, F = 3.
(a) Find a formula for F in terms of d.
(b) Find the value of d when F = 1.

(a) $F = \frac{k}{\sqrt{d}}$

Substitute d = 4 and F = 3: $3 = \frac{k}{\sqrt{4}}$

k = 6

$F = \frac{6}{\sqrt{d}}$

(b) $1 = \frac{6}{\sqrt{d}}$, so $\sqrt{d} = \frac{6}{1} = 6$, so d = 36

Answer: (a) $F = \frac{6}{\sqrt{d}}$ **(b) d = 36**

7 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 7 : 3.
Then 12 apples are removed and 10 pears are removed.
The ratio is now 3 : 1.
How many pears were there at the start?

Let the numbers at the start be 7k and 3k
(7k − 12) : (3k − 10) = 3 : 1, so 1(7k − 12) = 3(3k − 10)
7k − 12 = 9k − 30
−2k = −18, so k = 9
Number of pears at the start: 3 × 9 = 27
Answer: 27

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 16) and (3, 256).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 16$ and $k\,a^3 = 256$

Divide: $a^2 = \frac{256}{16} = 16$

a = 4 (positive, since a > 0)
From the first point: k × 4 = 16, so k = 4
Answer: k = 4, a = 4

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 6 cm, top radius 6 cm and base radius 15 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+6} = \frac{6}{15}$

15 h = 6(h + 6), so 9 h = 36 and h = 4

Large cone: $\frac{1}{3} \times \pi \times 15^2 \times 10 = 750\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 4 = 48\pi$

Frustum = $750\pi - 48\pi = 702\pi$ cm^3

Answer: 702π cm^3

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10 Surds and fractions

Simplify $\frac{1}{3+\sqrt{3}} + \frac{2}{3-\sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{3})(3-\sqrt{3}) = 9 - 3 = 6$

Numerator: $(3-\sqrt{3}) + 2(3+\sqrt{3})$

$= 9 + \sqrt{3}$

$\frac{9+\sqrt{3}}{6}$

Answer: $\mathbf{\frac{9+\sqrt{3}}{6}}$