Day 183 **S Tier 10-a-day: Model Solutions**

1 Winning a match

Tariq and Callum play a match. The first to win 2 table tennis games wins the match.

The probability that Tariq wins any one of the table tennis games is $\frac{3}{5}$.

Work out the probability that Tariq wins the match.

Tariq wins by WW (2 games) or WLW or LWW (3 games), where W means Tariq wins that game

P(WW) = $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

P(WLW) = P(LWW) = $\frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{18}{125}$

Total = $\frac{9}{25} + 2 \times \frac{18}{125} = \frac{81}{125}$

Answer: $\mathbf{\frac{81}{125}}$

2 Standard form

Work out $(9.3 \times 10^{-3}) \times (1.5 \times 10^{1})$

Give your answer in standard form.

$9.3 \times 1.5 = 13.95$

$10^{-3} \times 10^{1} = 10^{-2}$

$13.95 \times 10^{-2} = 1.395 \times 10^{-1}$

Answer: $\mathbf{1.395 \times 10^{-1}}$

3 Comparing costs

Zip Taxis has a fixed charge of 50p plus £5 per mile.

Go Cabs has a fixed charge of £3.50 plus £4.50 per mile.

(a) Work out the number of miles for which the two costs are the same.

(b) Which is cheaper for 12 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when 0.50 + 5n = 3.50 + 4.50n

0.50n = 3, so n = 6

(b) Zip Taxis: 50p + £5 × 12 = £60.50

Go Cabs: £3.50 + £4.50 × 12 = £57.50

Go Cabs is cheaper by £3

Answer: (a) 6 (b) Go Cabs, by £3

4 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 45°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 45°

Angle COB is an exterior angle of triangle OAC: 45 + 45 = 90°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 90°

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5 Direct and inverse proportion

F is inversely proportional to d.
When d = 4, F = 1.
(a) Find a formula for F in terms of d.
(b) Find the value of F when d = 2.

(a) $F = \frac{k}{d}$

Substitute d = 4 and F = 1: $1 = \frac{k}{4}$

k = 4

$F = \frac{4}{d}$

(b) $F = \frac{4}{2} = 2$

Answer: (a) $F = \frac{4}{d}$ (b) 2

6 Percentages of percentages

In a group of visitors, 75% are adults and the rest are children.
60% of the adults and 50% of the children buy a souvenir.
What percentage of all the visitors buy a souvenir?

Take 100 visitors: 75 are adults and 25 are children
60% of 75 = 45 and 50% of 25 = 12.5
45 + 12.5 = 57.5 out of 100
Answer: 57.5%

7 Conditional probability from a table

The table shows information about 95 employees.
(a) An employee who walks to work is chosen at random. Work out the probability that this employee is a man.
(b) Two of the 95 employees are chosen at random. Work out the probability that both are men.

(a) 38 employees are in the Walk column, and 25 of them are men

Probability = $\frac{25}{38}$

(b) There are 50 men, so P(both) = $\frac{50}{95} \times \frac{49}{94} = \frac{245}{893}$

Answer: (a) $\frac{25}{38}$ (b) $\frac{245}{893}$

8 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 6, 7, 15, 12
Mid-points: 12.5, 20, 32.5, 50
$\Sigma fm = 6\times12.5 + 7\times20 + 15\times32.5 + 12\times50 = 1302.5$, $\Sigma f = 40$
Mean = 1302.5 ÷ 40 = 32.6
Answer: 32.6

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9 Composite functions: fg and gf

f(x) = −x + 5 and g(x) = x^2 − 4
Solve fg(x) = gf(x)

fg(x) = f(x^2 − 4) = −x^2 + 9
gf(x) = g(−x + 5) = (−x + 5)^2 − 4 = x^2 − 10x + 21
Set them equal and rearrange: 2x^2 − 10x + 12 = 0
Divide by 2 and factorise: (x − 3)(x − 2) = 0
x = 2 or x = 3
Answer: x = 2 or x = 3

10 Cone made from a sector

A sector of a circle has radius 12 cm and angle 120°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{120}{360} \times 2\pi \times 12 = 8\pi$ cm
This is the circumference of the base: $2\pi r = 8\pi$, so r = 4 cm
(b) Slant height = 12 cm, so height^2 = $12^2 - 4^2$ = 128, height = $8\sqrt{2}$ cm
Volume = $\frac{1}{3} \times \pi \times 4^2 \times 8\sqrt{2} = \frac{128\sqrt{2}}{3}\pi$ cm^3

Answer: (a) 4 cm (b) $\frac{128\sqrt{2}}{3}\pi$ cm^3