Day 182 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

80 beetles in a field are caught, marked and put back.
Later, a second sample of 80 beetles is caught. 10 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 10 out of 80

This should match the proportion marked in the whole population: $\frac{10}{80} = \frac{80}{N}$

N = 80 × 80 ÷ 10 = 640

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 640 (b) The population stays the same between samples

2 HCF and LCM

The HCF of two numbers is 8 and their LCM is 96.
One of the numbers is 32.
Work out the other number.

HCF × LCM = product of the two numbers
8 × 96 = 768
Other number = 768 ÷ 32 = 24

Answer: 24

3 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 120 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mass greater than 25.

(a) Median at cumulative frequency 60, so median = 30
Lower quartile at 30, so LQ = 20. Upper quartile at 90, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 25, cumulative frequency = 45
Number greater = 120 − 45 = 75

Answer: (a) median 30, IQR 20 (b) 75

4 Standard form

Work out $(5.1 \times 10^{-5}) - (7.9 \times 10^{-8})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-8}: $5.1 \times 10^{-5} = 5100 \times 10^{-8}$
$(5100 - 7.9) \times 10^{-8} = 5092.1 \times 10^{-8}$
$= 5.0921 \times 10^{-5}$

Answer: 5.0921×10^{-5}

5 Algebraic fractions: adding and subtracting

Show that $\frac{6}{2x+5} + \frac{x-1}{2x+1}$ can be written as $\frac{2x^2+15x+1}{(2x+5)(2x+1)}$

The common denominator is (2x + 5)(2x + 1)

$$\frac{6}{2x+5} + \frac{x-1}{2x+1} = \frac{6(2x+1) + (x-1)(2x+5)}{(2x+5)(2x+1)}$$

Expand the numerator: $2x^2 + 15x + 1$

$$\frac{2x^2+15x+1}{(2x+5)(2x+1)}$$

Answer: $\frac{\mathbf{2x^2+15x+1}}{\mathbf{(2x+5)(2x+1)}}$

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6 Ratio and expressions

8x = 3y
Work out the ratio (6x + 2y) : (2y − 5x)
Give your answer in its simplest form.

8x = 3y gives x : y = 3 : 8, so let x = 3t and y = 8t
6(3t) + 2(8t) = 34t
2(8t) − 5(3t) = 1t
34t : 1t = 34 : 1
Answer: 34 : 1

7 Transformations of graphs

The graph of y = f(x) has a maximum point at (−3, 6).
Write down the coordinates of the turning point of the graph of
(a) y = f(x − 1)
(b) y = f(x − 3) + 1

(a) Start with the turning point (−3, 6)
Apply the transformation to the point: (−2, 6)
(b) f(x − 3) changes the x-coordinate to 0, and f(x) + 1 changes the y-coordinate to 7
Turning point: (0, 7)
Answer: (a) (−2, 6) (b) (0, 7)

8 Rationalising the denominator

Write $\frac{4+\sqrt{3}}{2+\sqrt{3}}$ in the form $p + q\sqrt{3}$
where p and q are rational numbers.

Multiply top and bottom by $(2 - \sqrt{3})$:
Numerator: $(4 + \sqrt{3})(2 - \sqrt{3}) = 5 - 2\sqrt{3}$
Denominator: 4 − 3 = 1
$= 5 - 2\sqrt{3}$
Answer: $5 - 2\sqrt{3}$

9 Direct and inverse proportion

F is directly proportional to the square of d.
When d = 8, F = 640.
(a) Find a formula for F in terms of d.
(b) Find the value of F when d = 4.

(a) $F = kd^2$
Substitute d = 8 and F = 640: $640 = k \times 8^2$
k = 10
$F = 10d^2$
(b) $F = 104^2 = 160$
Answer: (a) $F = 10d^2$ (b) 160

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10 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 4 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2

Answer: $\frac{1}{2}\pi$ cm^2