Day 181 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 37°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 37°
Angle COB is an exterior angle of triangle OAC: 37 + 37 = 74°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 74°

2 Surds

Given that $\sqrt{216} - \sqrt{24} = \sqrt{n}$,
find the value of n.

$\sqrt{216} = 6\sqrt{6}$ and $\sqrt{24} = 2\sqrt{6}$
$6\sqrt{6} - 2\sqrt{6} = 4\sqrt{6}$
$4\sqrt{6} = \sqrt{96}$, so n = 96
Answer: n = 96

3 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{5-0}{0-(-3)} = \frac{5}{3}$

Perpendicular gradient = $-1 \div \frac{5}{3} = -\frac{3}{5}$

M passes through B(−3, 0): $0 = (-\frac{3}{5}) \times (-3) + c$, so $c = -\frac{9}{5}$

$y = -\frac{3}{5}x - \frac{9}{5}$

Answer: $y = -\frac{3}{5}x - \frac{9}{5}$

4 Similar shapes

Two tins are mathematically similar.
The heights of the tins are 6 cm and 15 cm.
The smaller one has volume 80 cm^3.
Work out the volume of the larger one.

Scale factor of lengths = $15 \div 6 = \frac{5}{2}$

Scale factor of volumes = $(\frac{5}{2})^3 = \frac{125}{8}$

$80 \times \frac{125}{8} = 1250$ cm^3

Answer: 1250 cm^3

5 Factorising quadratics

Factorise $4x^2 + 16x + 7$

4 × 7 = 28. Two numbers that multiply to 28 and add to 16 are 14 and 2
Split the middle term: $4x^2 + 14x + 2x + 7$
Factorise in pairs: 2x(2x + 7) + (2x + 7)
= (2x + 1)(2x + 7)
Answer: (2x + 1)(2x + 7)

Day 181 **S Tier 10-a-day: Model Solutions**

6 Joint and inverse variation

F is directly proportional to the square of m and inversely proportional to d.
When m = 3 and d = 6, F = 36.
Work out the value of F when m = 6 and d = 2.

$F = \frac{km^2}{d}$

$36 = \frac{k \times 9}{6}$, so $k = 36 \times 6 \div 9 = 24$

$F = \frac{24 \times 36}{2} = 432$

Answer: F = 432

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (3, −4).
The curve crosses the y-axis at (0, 14).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (3, −4), so $y = a(x - 3)^2 - 4$
When x = 0, y = 14: $a \times 9 - 4 = 14$, so a = 2
$y = 2(x - 3)^2 - 4$
Expand: $y = 2x^2 - 12x + 14$
Answer: $y = 2x^2 - 12x + 14$

8 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 20 seconds to go completely through a tunnel which is 300 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 20 s = 25 × 20 = 500 m
This is the length of the tunnel plus the length of the train
Train = 500 − 300 = 200 m
Answer: 200 m

9 Rationalising the denominator

Write $\frac{5 + \sqrt{10}}{4 - \sqrt{10}}$ in the form $p + q\sqrt{10}$
where p and q are rational numbers.

Multiply top and bottom by $(4 + \sqrt{10})$:
Numerator: $(5 + \sqrt{10})(4 + \sqrt{10}) = 30 + 9\sqrt{10}$
Denominator: 16 − 10 = 6
$= \frac{10 + 3\sqrt{10}}{2}$

Answer: $\frac{10 + 3\sqrt{10}}{2}$

10 Density of an alloy

An alloy contains 219 g of tin and 27 g of aluminium.
The density of tin is 7.3 g/cm^3. The density of aluminium is 2.7 g/cm^3.
Work out the density of the alloy.

Volume of tin = 219 ÷ 7.3 = 30 cm^3
Volume of aluminium = 27 ÷ 2.7 = 10 cm^3
Total mass = 246 g, total volume = 40 cm^3
Density = 246 ÷ 40 = 6.15 g/cm^3
Answer: 6.15 g/cm^3