Day 180 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

FitLife has a joining fee of £7.50 plus £5 per month.
GymNow has a joining fee of £25.50 plus £2 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 11 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $7.50 + 5n = 25.50 + 2n$
$3n = 18$, so $n = 6$
(b) FitLife: £7.50 + £5 × 11 = £62.50
GymNow: £25.50 + £2 × 11 = £47.50
GymNow is cheaper by £15
Answer: (a) 6 (b) GymNow, by £15

2 Pressure

A wooden block in the shape of a cuboid weighs 930 N.
Its smallest face has area 100 cm^2. Its largest face has area 2,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 2,500 cm^2 = 0.25 m^2
(a) Greatest pressure is on the smallest face: 930 ÷ 0.01 = 93,000 N/m^2
(b) Least pressure is on the largest face: 930 ÷ 0.25 = 3,720 N/m^2
Answer: (a) 93,000 N/m^2 (b) 3,720 N/m^2

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (3x − 15)°, angle BCD = (5x + 35)° and angle ABC = (3x + 25)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(3x - 15) + (5x + 35) = 180$
$8x + 20 = 180$, so $8x = 160$
$x = 20$
(b) Angle ABC = 3 × 20 + 25 = 85°
Angle ADC = 180 − 85 = 95°
Answer: (a) x = 20 (b) 95°

4 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 38 is taken. Work out how many come from Group B.
(b) A different stratified sample has 24 from Group C. Work out the size of this sample.

Total = 110 + 100 + 120 + 50 = 380
(a) Group B: $\frac{100}{380} \times 38 = 10$
(b) Fraction sampled = 24 ÷ 120 = $\frac{1}{5}$, so sample size = $\frac{1}{5} \times 380 = 76$
Answer: (a) 10 (b) 76

5 Compound interest

Ivan puts £2,500 in a savings account.
Compound interest is paid at 2% in year 1, 2% in year 2 and 5% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.02, 1.02, 1.05
£2,500 × 1.02 × 1.02 × 1.05
= £2,731.05
Answer: £2,731.05

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6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on AB with AN : NB = 3 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{3}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a}) = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

(b) OM = $\frac{2}{5}\mathbf{a}$

MN = ON − OM = $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{2}{5}\mathbf{a} = \frac{3}{5}\mathbf{b}$

MN = $\frac{3}{5}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ (b) MN = $\frac{3}{5}\mathbf{b}$, parallel to OB

7 Simultaneous equations

Solve the simultaneous equations
$4x + 6y = 34$
$5x - y = 0$

To match the x terms: (1) × 5 gives $20x + 30y = 170$, and (2) × 4 gives $20x - 4y = 0$
Subtract the equations to eliminate x: $34y = 170$
$y = 5$
Substitute $y = 5$ into (1): $4x + 30 = 34$
$4x = 4$, so $x = 1$
Answer: $x = 1$, $y = 5$

8 Algebraic fractions: adding and subtracting

Show that $\frac{6}{x-4} - \frac{6}{x-2}$ can be written as $\frac{12}{(x-4)(x-2)}$

The common denominator is $(x - 4)(x - 2)$
$\frac{6}{x-4} - \frac{6}{x-2} = \frac{6(x-2) - 6(x-4)}{(x-4)(x-2)}$
Expand the numerator: 12
$\frac{12}{(x-4)(x-2)}$
Answer: $\frac{12}{(x-4)(x-2)}$

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9 Loci and regions

ABCD is a rectangle with AB = 2 cm and AD = 6 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 1 cm from A
AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{1}{2}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 1 cm, height $2\sin 60° = 1\sqrt{3}$ cm, area $= \frac{1}{2} \times 1 \times 1\sqrt{3} = \frac{1}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 2^2 = \frac{1}{3}\pi$

Area $= \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi$ cm^2

Answer: $(\frac{1}{2}\sqrt{3} + \frac{1}{3}\pi)$ cm^2

10 Bounds and accuracy

a = 0.483 and b = 27.8, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

0.4825 ≤ a < 0.4835; 27.75 ≤ b < 27.85
Lower bound = 0.4825 ÷ 27.85 = 0.017325...
Upper bound = 0.4835 ÷ 27.75 = 0.017423...
Both bounds round to 0.017 to 2 significant figures, but not to 3 significant figures
Answer: 0.017 (2 s.f.)