Day 18 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 2.
25% of the juniors and 55% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 60 students in total: 20 juniors and 40 seniors
25% of 20 = 5 and 55% of 40 = 22
Together: 27 out of 60
27 ÷ 60 × 100 = 45%
Answer: 45%

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 63°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 63 = 126°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 126 = 54°
Answer: (a) 126° (b) 54°

3 Multiplying and dividing surds

Simplify fully $\frac{4\sqrt{112}}{2\sqrt{7}}$

$\sqrt{112} = 4\sqrt{7}$
$\frac{4\sqrt{112}}{2\sqrt{7}} = \frac{16\sqrt{7}}{2\sqrt{7}}$
= 8
Answer: 8

4 Using a line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The equation of the line of best fit is y = 2.5x + 8.
(a) Estimate y when x = 3.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) y = 2.5 × 3 + 8 = 15.5
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 15.5 (b) x = 15 is outside the range of the data

5 Comparing costs

Zip Taxis has a fixed charge of £1.50 plus £4 per mile.
Go Cabs has a fixed charge of £16.50 plus £1.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 13 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when 1.50 + 4n = 16.50 + 1.50n
2.50n = 15, so n = 6
(b) Zip Taxis: £1.50 + £4 × 13 = £53.50
Go Cabs: £16.50 + £1.50 × 13 = £36
Go Cabs is cheaper by £17.50
Answer: (a) 6 (b) Go Cabs, by £17.50

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6 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 41°} = \frac{9}{\sin 49°}$

$x = \frac{9 \times \sin 41°}{\sin 49°} = 7.8236$

x = 7.8 cm (1 d.p.)

Answer: AC = 7.8 cm

7 LCM problems

A red light flashes every 7 minutes.
A blue light flashes every 10 minutes.
A green light flashes every 14 minutes.
They all flash together at 7:00 am.
How many times do they all flash together between 7:00 am and 3:00 pm, including both 7:00 am and 3:00 pm if they do?

Find the LCM of 7, 10 and 14
7 = 7, 10 = 2 × 5, 14 = 2 × 7
LCM = 2 × 5 × 7 = 70
Total time from 7:00 am to 3:00 pm = 480 minutes
480 ÷ 70 = 6.86, so there are 6 more times after the start
Including the start: 6 + 1 = 7
Answer: 7

8 Ratio and money transferred

Kira and Nadia share some money in the ratio 7 : 1.
Kira gives Nadia £55.
The ratio of their money is now 1 : 3.
How much money do they have altogether?

Let the amounts be 7k and 1k, so the total is 8k

Kira now has 7k − 55, which is $\frac{1}{4}$ of the total

4(7k − 55) = 1 × 8k
28k − 220 = 8k, so 20k = 220 and k = 11
Total = 8 × 11 = £88
Answer: £88

9 Algebraic proof (harder)

Prove algebraically that the sum of the cubes of any three consecutive integers is a multiple of 9.

$(m - 1)^3 + m^3 + (m + 1)^3 = 3m^3 + 6m = 3m(m^2 + 2)$

If m is a multiple of 3, then $3m(m^2 + 2)$ is a multiple of 9

If $m = 3m + 1$ or $m = 3m - 1$, then $m^2 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$, a multiple of 3

In every case $3m(m^2 + 2)$ contains two factors of 3, so it is a multiple of 9

Answer: Proved

10 Arc length and sector area

A sector of a circle has radius 5 cm and angle 120°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{120}{360}$

Arc length = $\frac{120}{360} \times 2 \times \pi \times 5 = \frac{10}{3}\pi$ cm

(b) Area = $\frac{120}{360} \times \pi \times 5^2 = \frac{25}{3}\pi$ cm^2

Answer: (a) $\frac{10}{3}\pi$ cm (b) $\frac{25}{3}\pi$ cm^2