Day 179 **S Tier 10-a-day: Model Solutions**

1 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 6 cm and the area of triangle ADE is 96 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 6 = 10 cm

Length scale factor = $10 \div 4 = \frac{5}{2}$

Area scale factor = $(\frac{5}{2})^2 = \frac{25}{4}$

Area of triangle ABC = $96 \times \frac{25}{4} = 600$ cm^2

Trapezium = $600 - 96 = 504$ cm^2

Answer: 504 cm^2

2 Quadratic inequalities

Solve $x^2 \leq -11x - 28$

Rearrange so that one side is 0: $x^2 + 11x + 28 \leq 0$
Solve $x^2 + 11x + 28 = 0$: $(x + 7)(x + 4) = 0$, so $x = -7$ or $x = -4$
The graph is a U shape crossing the x-axis at -7 and -4; it is below the axis between the roots
$-7 \leq x \leq -4$
Answer: $-7 \leq x \leq -4$

3 Histograms with an unmarked scale

The histogram shows the lengths of leaves collected from a tree.
400 of them had a length between 60 and 80 cm.
(a) How many had a length between 30 and 40 cm?
(b) How many leaves are there in total?

Frequency is proportional to the area of a bar
60 to 80: bar is 4 squares high and 20 wide, area 80, which represents 400
So 1 unit of area represents $400 \div 80 = 5$
(a) 30 to 40: $5 \times 10 = 50$ units of area, so $50 \times 5 = 250$
(b) Total area = 20 + 120 + 50 + 160 + 80 = 430, so total = $430 \times 5 = 2150$
Answer: (a) 250 (b) 2150

4 Percentage change

The price of a phone is decreased by 10%.
The new price is then increased by 50%.
Work out the overall percentage change in the price.

Multipliers: 0.9 and 1.5
Overall multiplier = $0.9 \times 1.5 = 1.35$
$1.35 - 1 = 0.35$, so an overall increase of 35%
Answer: 35% increase

5 Combined transformations

The point P(6, −1) is rotated 180° about the origin.
The image is then reflected in the y-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−6, 1)
After the second transformation: P'' = (6, 1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
Answer: (a) (6, 1) (b) Reflection in the x-axis

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6 Relative speed

Nadia sets off at 10:00 and cycles at 16 km/h along a path.
Isla sets off from the same place at 10:30 and cycles at 36 km/h along the same path.
At what time does Isla catch up with Nadia?

When Isla starts, Nadia is 0.5 × 16 = 8 km ahead
Isla gains 36 − 16 = 20 km every hour
Time to catch up = 8 ÷ 20 = 0.4 hours = 24 minutes
10:30 + 24 minutes = 10:54
Answer: 10:54

7 Bounds and accuracy

a = 9.3 and b = 7.12, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$9.295 \leq a < 9.305$; $7.115 \leq b < 7.125$
Lower bound = 9.295 × 7.115 = 66.133925
Upper bound = 9.305 × 7.125 = 66.298125
Both bounds round to 66 to 2 significant figures, but not to 3 significant figures
Answer: 66 (2 s.f.)

8 Simple and compound interest

Mo invests £2,000 for 4 years at 1.5% per year simple interest.
Nia invests £2,000 for 4 years at 1.5% per year compound interest.
Work out how much more interest Nia earns than Mo.
Give your answer to the nearest penny. (Calculator)

Simple interest: £2,000 × 1.5% × 4 = £120
Compound: £2,000 × 1.015^4 = £2,122.73
Compound interest = £2,122.73 − £2,000 = £122.73
Difference = £122.73 − £120 = £2.73
Answer: £2.73

9 Loci and regions

ABCD is a rectangle with AB = 9 cm and AD = 10 cm.
A point P inside the rectangle is such that AP ≤ 6 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 6 cm: P is inside the quarter circle of radius 6 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 6^2 = \frac{9}{2}\pi$ cm^2
Answer: $\frac{9}{2}\pi$ cm^2

10 Algebraic proof (harder)

Prove that the sum of the cubes of any three consecutive integers is a multiple of 9.

$(m-1)^3 + m^3 + (m+1)^3 = 3m^3 + 6m = 3m(m^2 + 2)$
If m is a multiple of 3, then $3m(m^2 + 2)$ is a multiple of 9
If $m = 3m + 1$ or $m = 3m - 1$, then $m^2 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$, a multiple of 3
In every case $3m(m^2 + 2)$ contains two factors of 3, so it is a multiple of 9
Answer: Proved