**Day 178** **S Tier 10-a-day: Model Solutions**

## 1 Powers of standard form

Work out $(3.6 \times 10^{-6})^3$
Give your answer in standard form.

$(3.6 \times 10^{-6})^3 = 3.6^3 \times (10^{-6})^3$
$3.6^3 = 46.656$ and $(10^{-6})^3 = 10^{-18}$
$46.656 \times 10^{-18} = 4.6656 \times 10^{-17}$
**Answer: $4.6656 \times 10^{-17}$**

## 2 Area of a triangle using sine

Triangle ABC has AB = 7 cm and BC = 8 cm.
The area of triangle ABC is 14 $cm^2$.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area $= \frac{1}{2} \times AB \times BC \times \sin B$

$14 = \frac{1}{2} \times 7 \times 8 \times \sin B$

$\sin B = 14 \div 28 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is obtuse, so $B = 180° - 30° = 150°$

**Answer: 150°**

## 3 Standard form

Work out $(1.9 \times 10^{-3}) \times (6.4 \times 10^{-8})$
Give your answer in standard form.

$1.9 \times 6.4 = 12.16$
$10^{-3} \times 10^{-8} = 10^{-11}$
$12.16 \times 10^{-11} = 1.216 \times 10^{-10}$
**Answer: $1.216 \times 10^{-10}$**

## 4 Quadratic graphs

The curve $y = x^2 + 8x - 9$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 + 8x - 9 = 0$, so $(x + 9)(x - 1) = 0$
$x + 9 = 0$ or $x - 1 = 0$, so $x = -9$ or $x = 1$
(b) The turning point is halfway between A and B: $x = (-9 + 1) \div 2 = -4$
When $x = -4$: $y = (-4)^2 + 8 \times (-4) - 9 = -25$
**Answer: (a) A(−9, 0), B(1, 0) (b) (−4, −25)**

## 5 Speed-time graphs

The graph shows the speed of a car during 15 seconds.
(a) Work out the acceleration during the first 3 seconds.
(b) Work out the average speed over the whole 15 seconds.

(a) Acceleration = change in speed ÷ time = $(12 - 0) \div 3 = 4$ $m/s^2$
(b) Distance = area under the graph
Trapezium: $\frac{0 + 12}{2} \times 3 = 18$; rectangle: $12 \times 5 = 60$; triangle: $\frac{1}{2} \times 7 \times 12 = 42$
Total = 120 m, so average speed = $120 \div 15 = 8$ m/s
**Answer: (a) 4 $m/s^2$ (b) 8 m/s**

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## 6 HCF and LCM

(a) Write 35 and 140 as products of prime factors.
(b) Find the HCF and the LCM of 35 and 140.
Give your answers as numbers.

(a) $35 = 5 \times 7$ and $140 = 2^2 \times 5 \times 7$
(b) HCF $= 5 \times 7 = 35$
LCM $= 2^2 \times 5 \times 7 = 140$
**Answer: (a) $5 \times 7$; $2^2 \times 5 \times 7$ (b) HCF = 35, LCM = 140**

## 7 Estimating the median from grouped data

The table gives information about 98 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 10, 21, 32, 57, 82, 98
The median is the value at position $98 \div 2 = 49$
It is in the class 15 to 20, with 32 values before it, so we need 17 more of the 25 values in this class
Median $\approx 15 + \frac{17}{25} \times 5 = 18.4$
**Answer: 18.4**

## 8 Winning a match

Tariq and Hassan play a match. The first to win 2 table tennis games wins the match.
The probability that Tariq wins any one of the table tennis games is $\frac{1}{3}$.
Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two
P(Tariq wins then loses) $= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
P(Tariq loses then wins) $= \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
Total $= 2 \times \frac{2}{9} = \frac{4}{9}$
**Answer: $\frac{4}{9}$**

## 9 Iteration

Use the iteration formula $x_{n+1} = \sqrt{7x_n + 6}$ with $x_0 = 1$ (Calculator)
to work out $x_1$, $x_2$ and $x_3$.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{7 \times 1 + 6} = 3.60555...$
$x_2 = \sqrt{7 \times x_1 + 6} = 5.58917...$
$x_3 = \sqrt{7 \times x_2 + 6} = 6.71746...$
Rounded: $x_1 = 3.606$, $x_2 = 5.589$, $x_3 = 6.717$
**Answer: $x_1 = 3.606$, $x_2 = 5.589$, $x_3 = 6.717$**

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## 10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{a} + \mathbf{b}$, OQ = $10\mathbf{a} - 2\mathbf{b}$ and OR = $12\mathbf{a} - 3\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $6\mathbf{a} - 3\mathbf{b}$
QR = OR − OQ = $2\mathbf{a} - \mathbf{b}$

QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 1
**Answer: (b) 3 : 1**