Day 177 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The sum of the interior angles of a polygon is 2340°.
Work out the number of sides of the polygon.

Sum of interior angles = $(n - 2) \times 180°$
$(n - 2) \times 180 = 2340$
$n - 2 = 2340 \div 180 = 13$
$n = 15$
Answer: 15

2 Distance-time graphs

The graph shows Nadia's cycle ride to a café and back.
(a) Work out Nadia's speed on the way back.
(b) Work out Nadia's average speed for the whole ride, including the stop.

(a) Return: 40 km in 5 hours, so speed = $40 \div 5 = 8$ km/h
(b) Total distance = $2 \times 40 = 80$ km, total time = 8 hours
Average speed = $80 \div 8 = 10$ km/h
Answer: (a) 8 km/h (b) 10 km/h

3 Enlargement

Triangle T has vertices A(0, −1), B(−4, 1), C(−4, −3).
T is enlarged with scale factor $\frac{3}{2}$ and centre (−2, −3).
(a) Write down the coordinates of the image of A.
(b) The area of T is 15 square units. Work out the area of the image of T.

(a) Vector from the centre to A = $(0 - (-2), -1 - (-3)) = (2, 2)$
Multiply by $\frac{3}{2}$: (3, 3)
Image of A = centre + this vector = (1, 0)
(b) Area scale factor = $(\frac{3}{2})^2 = \frac{9}{4}$
Area = $15 \times \frac{9}{4} = \frac{135}{4}$ square units
Answer: (a) (1, 0) (b) $\frac{135}{4}$

4 Rates of work

A tank is filled by a tap in 12 minutes.
A drain can empty the full tank in 20 minutes.
The tap and the drain are both open, and the tank is empty to begin with.
How long does it take for the tank to fill?

In 1 minute: the tap fills $\frac{1}{12}$ and the drain empties $\frac{1}{20}$
Net: $\frac{1}{12} - \frac{1}{20} = \frac{8}{240}$
Time = $\frac{240}{8} = 30$ minutes
Answer: 30 minutes

5 Surds

Given that $\sqrt{18} - \sqrt{2} = \sqrt{n}$,
find the value of n.

$\sqrt{18} = 3\sqrt{2}$
$3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$
$2\sqrt{2} = \sqrt{8}$, so n = 8
Answer: n = 8

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6 Speed-time graphs

The graph shows the speed of a cyclist during a journey.
(a) Work out the deceleration in the last 4 seconds.
(b) Work out the average speed over the whole journey.

(a) Deceleration = $16 \div 4 = 4$ m/s^2

(b) Total distance = area under the graph = $\frac{1}{2} \times 8 \times 16 + 16 \times 12 + \frac{1}{2} \times 4 \times 16 = 288$ m

Average speed = total distance ÷ total time = $288 \div 24 = 12$ m/s

Answer: (a) 4 m/s^2 (b) 12 m/s

7 Percentages of percentages

In a group of students, 30% are boys and the rest are girls.
60% of the boys and 10% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 30 are boys and 70 are girls
60% of 30 = 18 and 10% of 70 = 7
18 + 7 = 25 out of 100
Answer: 25%

8 Tangent to a circle

A circle has equation $(x - 2)^2 + (y - 2)^2 = 50$.
The point P(9, 3) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (2, 2). The gradient of the radius to P is $\frac{3 - 2}{9 - 2} = \frac{1}{7}$

The tangent is perpendicular to the radius, so its gradient is -7. Using P: $y - 3 = -7(x - 9)$
Expand and rearrange: $7x + y = 66$
Answer: 7x + y = 66

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 6 cm and base radius 12 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h + 9} = \frac{6}{12}$

$12h = 6(h + 9)$, so $6h = 54$ and $h = 9$

Large cone: $\frac{1}{3} \times \pi \times 12^2 \times 18 = 864\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 9 = 108\pi$

Frustum = $864\pi - 108\pi = 756\pi$ cm^3

Answer: 756π cm^3

10 Quadratic inequalities

Solve $3x^2 \leq 15x + 18$

Rearrange so that one side is 0: $3x^2 - 15x - 18 \leq 0$
Divide by 3: $x^2 - 5x - 6 \leq 0$
Solve $x^2 - 5x - 6 = 0$: $(x + 1)(x - 6) = 0$, so $x = -1$ or $x = 6$
The graph is a U shape crossing the x-axis at -1 and 6; it is below the axis between the roots
$-1 \leq x \leq 6$
Answer: $-1 \leq x \leq 6$