Day 176 **S Tier 10-a-day: Model Solutions**

1 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 2) cm, DE = 6 cm and BC = 14 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 2} = \frac{6}{14}$
$14x = 6(2x + 2)$
$14x = 12x + 12$
$2x = 12$
$x = 6$
(b) $AB = 2x + 2 = 14$ cm
Answer: (a) x = 6 (b) 14 cm

2 Capture and recapture

30 deer in a forest are caught, marked and put back.
Later, a second sample of 40 deer is caught. 5 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 40

This should match the proportion marked in the whole population: $\frac{5}{40} = \frac{30}{N}$

$N = 30 \times 40 \div 5 = 240$
(b) No deer are born, die or leave between the samples, and the deer mix evenly
Answer: (a) 240 (b) The population stays the same between samples

3 Powers of standard form

Work out $(3.6 \times 10^2)^3$
Give your answer in standard form.

$(3.6 \times 10^2)^3 = 3.6^3 \times (10^2)^3$
$3.6^3 = 46.656$ and $(10^2)^3 = 10^6$
$46.656 \times 10^6 = 4.6656 \times 10^7$
Answer: 4.6656×10^7

4 Distance-time graphs

The graph shows Maya's cycle ride to a café and back.
(a) Work out Maya's speed on the way back.
(b) Work out Maya's average speed for the whole ride, including the stop.

(a) Return: 12 km in 1.5 hours, so speed = 12 ÷ 1.5 = 8 km/h
(b) Total distance = 2 × 12 = 24 km, total time = 3 hours
Average speed = 24 ÷ 3 = 8 km/h
Answer: (a) 8 km/h (b) 8 km/h

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5 Perpendicular bisector

A is the point (−3, −4) and B is the point (3, 4).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (0, 0)

Gradient of AB = $\frac{4}{3}$

Perpendicular gradient = $-\frac{3}{4}$

$y - 0 = (-\frac{3}{4})(x - 0)$

Rearrange to integer coefficients: 3x + 4y = 0

Answer: 3x + 4y = 0

6 Compound interest

Fatima invests £8,500. The rates of interest for the next 3 years are 2%, 3% and 6%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.02, 1.03 and 1.06
8500 × 1.02 × 1.03 × 1.06 = 9465.91
To the nearest penny: £9,465.91
Answer: £9,465.91

7 Geometric sequences

The 2nd term of a geometric sequence is 24.
The 4th term of the sequence is 54.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 54 \div 24 = \frac{9}{4}$

$r = \sqrt{\frac{9}{4}} = \frac{3}{2}$ (positive, since all terms are positive)

(b) The 2nd term is $a \times r^1$: $a \times \frac{3}{2} = 24$

$a = 24 \div \frac{3}{2} = 16$

Answer: (a) r = $\frac{3}{2}$ (b) 16

8 Profit and loss

A retailer buys a bike and adds a 50% mark-up to get the selling price.
In a sale the selling price is reduced by 25% to £427.50.
Work out how much the shop paid for a bike.

Overall multiplier = 1.5 × 0.75 = 1.125
Cost = £427.50 ÷ 1.125
= £380
Answer: £380

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9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $\mathbf{a} + 5\mathbf{b}$, OQ = $4\mathbf{a} + 11\mathbf{b}$ and OR = $8\mathbf{a} + 19\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $3\mathbf{a} + 6\mathbf{b}$
QR = OR − OQ = $4\mathbf{a} + 8\mathbf{b}$

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4
Answer: (b) 3 : 4

10 Equation of a circle

A and B are the points (0, 3) and (−8, −9).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{0 + (-8)}{2}, \frac{3 + (-9)}{2}) = (-4, -3)$

$r^2 = (0 - (-4))^2 + (3 - (-3))^2 = 16 + 36 = 52$

$(x + 4)^2 + (y + 3)^2 = 52$

Answer: $(x + 4)^2 + (y + 3)^2 = 52$