Day 175 **S Tier 10-a-day: Model Solutions**

1 Multiplying and dividing surds

Work out $(3\sqrt{6})^2 - (5\sqrt{6})^2$

$(3\sqrt{6})^2 = 3^2 \times 6 = 54$
$(5\sqrt{6})^2 = 5^2 \times 6 = 150$
$54 - 150 = -96$
Answer: -96

2 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -2$ and $x = 3$, so $y = a(x + 2)(x - 3)$
When $x = 0$, $y = -6$: $a \times 2 \times (-3) = -6$, so $a = 1$
$y = (x + 2)(x - 3)$
$y = x^2 - x - 6$
Answer: $y = x^2 - x - 6$

3 Similar shapes

Two shapes are mathematically similar.
The smaller has height 40 cm and area 36 cm^2.
The larger has area 12.96 cm^2.
Work out the height of the larger shape.

Ratio of areas = 12.96 : 36 = 9 : 25
Ratio of lengths = 3 : 5
Height = $40 \times \frac{3}{5} = 24$ cm
Answer: 24 cm

4 Averages with algebra

Here are five numbers: $x + 1$, $x + 2$, $3x + 1$, 3, $x - 1$.
The mean of the five numbers is 12.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(x + 1) + (x + 2) + (3x + 1) + (3) + (x - 1) = 12 \times 5 = 60$
$6x + 6 = 60$, so $6x = 54$ and $x = 9$
The numbers are 10, 11, 28, 3, 8
(b) Range = $28 - 3 = 25$
Answer: (a) x = 9 (b) 25

5 Quadratic graphs

The curve $y = 2x^2 - 12x$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$. $2x^2 - 12x = 0$, so $2(x^2 + 6x) = 0$, giving $2(x)(x - 6) = 0$
$x = 0$ or $x - 6 = 0$, so $x = 0$ or $x = 6$
(b) The turning point is halfway between A and B: $x = (0 + 6) \div 2 = 3$
When $x = 3$: $y = 2 \times 3^2 - 12 \times 3 + 0 = -18$
Answer: (a) A(0, 0), B(6, 0) (b) (3, −18)

Day 175 **S Tier 10-a-day: Model Solutions**

6 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 12^2 + 14^2 = 340$
$AG^2 = AC^2 + CG^2 = 340 + 12^2 = 484$
AG = 22 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{12}{\sqrt{340}}$

angle GAC = 33.1° (1 d.p.)
Answer: (a) 22 cm (b) 33.1°

7 Histograms

The histogram shows the lengths of leaves collected from a tree.
(a) Work out the total number of leaves.
(b) Estimate the number of leaves with a length between 15 cm and 50 cm.

Frequency = frequency density × class width
Multiply each height by its class width: 20, 10, 40, 32, 16
(a) Total = 118
(b) 15 to 20: 1 × 5 = 5
Whole bar between: 40
40 to 50: 1.6 × 10 = 16, so estimate = 61
Answer: (a) 118 (b) 61

8 Bounds of speed, density and pressure

A runner covers a distance of 3300 m, correct to the nearest 100.
The race takes 200 s, correct to the nearest 5.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

3250 ≤ distance < 3350; 197.5 ≤ time < 202.5
Lower bound of speed = smallest distance ÷ biggest time = 3250 ÷ 202.5
= 16.0494... = 16.0 (3 s.f.)
Answer: 16.0 m/s

9 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 41$
$x + y = -1$

From the second equation, $y = -x - 1$
Substitute into the first equation: $x^2 + (-x - 1)^2 = 41$
Expand: $2x^2 + 2x - 40 = 0$
Divide by 2: $x^2 + x - 20 = 0$
Factorise: $(x + 5)(x - 4) = 0$, so $x = -5$ or $x = 4$
$y = -x - 1$ gives $y = 4$ and $y = -5$
Answer: $x = -5$, $y = 4$ and $x = 4$, $y = -5$

Day 175 **S Tier 10-a-day: Model Solutions**

10 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 3 : 4.
Tin B holds red paint and white paint in the ratio 1 : 6.
42 litres from tin A and 21 litres from tin B are mixed.
Work out the ratio of red paint to white paint in the mixture.

From A: red paint = $42 \times \frac{3}{7}$ = 18 litres, white paint = 24 litres

From B: red paint = $21 \times \frac{1}{7}$ = 3 litres, white paint = 18 litres

red paint: 18 + 3 = 21 and white paint: 24 + 18 = 42
21 : 42 = 1 : 2
Answer: 1 : 2