Day 174 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 19 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 19e.
Interior angle + exterior angle = 180°, so 19e + e = 180°
e = 180° ÷ 20 = 9°
Number of sides = 360° ÷ 9° = 40
Answer: 40

2 Recurring decimals

Show that $0.4\dot{9}7\dot{6} = \frac{2486}{4995}$

Let x = 0.497697...
10x = 4.976976...
10,000x = 4976.976976...
Subtract: 9990x = 4972
$x = \frac{4972}{9990} = \frac{2486}{4995}$
Answer: x = $\frac{2486}{4995}$

3 Arc length and sector area

A sector of a circle has radius 10 cm and angle 45°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{45}{360}$
Arc length = $\frac{45}{360} \times 2 \times \pi \times 10 = \frac{5}{2}\pi$ cm
(b) Area = $\frac{45}{360} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2
Answer: (a) $\frac{5}{2}\pi$ cm (b) $\frac{25}{2}\pi$ cm^2

4 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (4, −7).
The curve crosses the y-axis at (0, 25).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (4, −7), so $y = a(x - 4)^2 - 7$
When x = 0, y = 25: a × 16 − 7 = 25, so a = 2
$y = 2(x - 4)^2 - 7$
Expand: $y = 2x^2 - 16x + 25$
Answer: $y = 2x^2 - 16x + 25$

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5 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 68° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{7} = \frac{\sin 68°}{12}$

$\sin\theta = \frac{7 \times \sin 68°}{12} = 0.54086$

$\theta = \sin^{-1}(0.54086) = 32.7°$ (1 d.p.)

Answer: $\theta = 32.7°$

6 Standard form

Work out $(4.5 \times 10^{-5}) - (7.8 \times 10^{-8})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-8}: $4.5 \times 10^{-5} = 4500 \times 10^{-8}$

$(4500 - 7.8) \times 10^{-8} = 4492.2 \times 10^{-8}$

$= 4.4922 \times 10^{-5}$

Answer: 4.4922×10^{-5}

7 Combining means

The mean of the marks in a test for the Year 10 students is 72.
The mean of the marks in a test for the Year 11 students is 66.
The mean for all of the Year 10 students and Year 11 students together is 71.
Work out the ratio of the number of Year 10 students to the number of Year 11 students.

Let there be p Year 10 students and q Year 11 students

Total = 72p + 66q, and the overall mean gives $72p + 66q = 71(p + q)$

$1p = 5q$

$p : q = 5 : 1$

Answer: 5 : 1

8 Prime factors and indices

Write $12^2 \times 36^3$ as a product of powers of prime numbers.

$12 = 2^2 \times 3$, $36 = 2^2 \times 3^2$

$12^2 = 2^4 \times 3^2$

$36^3 = 2^6 \times 3^6$

Add the powers of each prime: $2^{10} \times 3^8$

Answer: $2^{10} \times 3^8$

9 Tangent to a circle: proof

Show that the line $y = -2x + 20$ is a tangent to the circle $x^2 + y^2 = 80$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -2x + 20$ into $x^2 + y^2 = 80$: $x^2 + (-2x + 20)^2 = 80$

Expand: $5x^2 - 80x + 320 = 0$

$b^2 - 4ac = (-80)^2 - 4 \times 5 \times 320 = 6400 - 6400 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-80)}{2 \times 5} = 8$, and $y = -2 \times 8 + 20 = 4$. The point is (8, 4)

Answer: (8, 4)

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10 Inverse proportion in context

9 workers can build a wall in 15 days.
After 3 days, 6 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 9 × 15 = 135 worker-days
Work done = 9 × 3 = 27, so work left = 108
Workers left = 9 − 6 = 3
Time = 108 ÷ 3 = 36 days
Answer: 36 days