Day 173 **S Tier 10-a-day: Model Solutions**

1 Factorising fully

Factorise fully $x^4 - 16$

$x^4 - 16 = (x^2)^2 - 4^2 = (x^2 - 4)(x^2 + 4)$

$x^2 - 4$ is also a difference of two squares: $(x - 2)(x + 2)$

Answer: $(x^2 + 4)(x - 2)(x + 2)$

2 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £25,000 is taxed at 20%.
All income above that is taxed at 40%.
Mo earns £48,500 in a year.
Work out how much income tax Mo pays.

Taxed at 20%: £25,000
20% of £25,000 = £5,000
Taxed at 40%: £48,500 − £15,000 − £25,000 = £8,500
40% of £8,500 = £3,400
Total tax = £5,000 + £3,400 = £8,400
Answer: £8,400

3 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 29 is taken. Work out how many come from Group B.
(b) A different stratified sample has 9 from Group D. Work out the size of this sample.

Total = 240 + 140 + 140 + 60 = 580

(a) Group B: $\frac{140}{580} \times 29 = 7$

(b) Fraction sampled = $9 \div 60 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 580 = 87$

Answer: (a) 7 (b) 87

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(2x + 30)°$, angle BCD = $(3x + 50)°$ and angle ABC = $(3x + 30)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(2x + 30) + (3x + 50) = 180$
$5x + 80 = 180$, so $5x = 100$
$x = 20$
(b) Angle ABC = $3 \times 20 + 30 = 90°$
Angle ADC = $180 - 90 = 90°$
Answer: (a) x = 20 (b) 90°

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt{7x_n + 12}$ with $x_0 = 3$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{7 \times 3 + 12} = 5.74456...$

$x_2 = \sqrt{7 \times x_1 + 12} = 7.22578...$

$x_3 = \sqrt{7 \times x_2 + 12} = 7.91078...$

Rounded: $x_1 = 5.745$, $x_2 = 7.226$, $x_3 = 7.911$

Answer: $x_1 = 5.745$, $x_2 = 7.226$, $x_3 = 7.911$

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6 Proof with indices

Show that $2^{m+2} - 2^m$ is a multiple of 3 for all integer values of m.

$2^{m+2} = 2^m \times 2^2$

$2^{m+2} - 2^m = 2^m(2^2 - 1)$

$2^2 - 1 = 3$

$= 3 \times 2^m$, which is a multiple of 3

Answer: 3×2^m

7 Comparing box plots

The box plots show the times taken by two groups, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 60 − 25 = 35
IQR of B = 80 − 70 = 10
(b) Group A has the lower median (45 compared with 75), so on average Group A was faster
Group B has the smaller interquartile range (10 compared with 35), so its times are more consistent
Answer: (a) A: 35, B: 10 (b) A has the lower median, B has the smaller IQR

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 6) and (4, 48).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 6$ and $k\,a^4 = 48$

Divide: $a^3 = \frac{48}{6} = 8$

$a = 2$
From the first point: $k \times 2 = 6$, so $k = 3$
Answer: $k = 3$, $a = 2$

9 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £12.00 per kg and type B costs £5.00 per kg.
They are mixed in the ratio A : B = 3 : 2.
The mixture is sold for £13.80 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 3 × £12.00 + 2 × £5.00 = £46.00
Cost per kg = £46.00 ÷ 5 = £9.20
Profit per kg = £13.80 − £9.20 = £4.60
Percentage profit = 4.60 ÷ 9.20 × 100 = 50%
Answer: 50%

10 Loci and regions

ABCD is a rectangle with AB = 7 cm and AD = 7 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 4^2 = 2\pi$ cm^2

Answer: 2π cm^2