Day 172 **S Tier 10-a-day: Model Solutions**

1 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-3\mathbf{a} + 5\mathbf{b}$, OQ = $7\mathbf{a} + 7\mathbf{b}$ and OR = $27\mathbf{a} + 11\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $10\mathbf{a} + 2\mathbf{b}$
QR = OR − OQ = $20\mathbf{a} + 4\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 4 = 1 : 2
Answer: (b) 1 : 2

2 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (4, 38) and (6, 37).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{37 - 38}{6 - 4} = \frac{-1}{2} = -0.5$
y = −0.5x + c: using (4, 38), 38 = −0.5 × 4 + c, so c = 40
y = −0.5x + 40
(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5
Answer: (a) y = −0.5x + 40 (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5

3 Percentage change

The price of a phone is increased by 10%.
The new price is then decreased by 25%.
Work out the overall percentage change in the price.

Multipliers: 1.1 and 0.75
Overall multiplier = 1.1 × 0.75 = 0.825
0.825 − 1 = −0.175, so an overall decrease of 17.5%
Answer: 17.5% decrease

4 Venn diagrams with algebra

The Venn diagram shows how many of 19 students have a tablet (T) or a laptop (L).
(a) Work out the value of x.
(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: (x + 1) + x + (2x − 1) + 3 = 19
4x + 3 = 19, so x = 4
(b) Students in the second circle: x + 2x − 1 = 4 + 7 = 11, of whom 4 are in both
Probability = $\frac{4}{11}$
Answer: (a) x = 4 (b) $\frac{4}{11}$

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt{2x_n + 2}$ with $x_0 = 3$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{2 \times 3 + 2} = 2.82843...$
$x_2 = \sqrt{2 \times x_1 + 2} = 2.76710...$
$x_3 = \sqrt{2 \times x_2 + 2} = 2.74485...$
Rounded: $x_1 = 2.828$, $x_2 = 2.767$, $x_3 = 2.745$
Answer: $x_1 = 2.828$, $x_2 = 2.767$, $x_3 = 2.745$

Day 172 **S Tier 10-a-day: Model Solutions**

6 Simple and compound interest

Nia invests £1,000 for 3 years at 3.5% per year simple interest.
Hana invests £1,000 for 3 years at 3.5% per year compound interest.
Work out how much more interest Hana earns than Nia.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 3.5% × 3 = £105

Compound: £1,000 × 1.035^3 = £1,108.72

Compound interest = £1,108.72 − £1,000 = £108.72

Difference = £108.72 − £105 = £3.72

Answer: £3.72

7 Perpendicular bisector

A is the point (−4, −2) and B is the point (−10, 2).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−7, 0)

Gradient of AB = $-\frac{2}{3}$

Perpendicular gradient = $\frac{3}{2}$

$y - 0 = \frac{3}{2}(x - (-7))$

Rearrange to integer coefficients: 3x − 2y = −21

Answer: 3x − 2y = −21

8 Squares, cubes and prime factors

Find the smallest positive integer k such that 225 × k is a cube number.

$225 = 3^2 \times 5^2$

For a cube number every power must be a multiple of 3

Multiply by 3 × 5 to get $3^3 \times 5^3$

k = 15

Answer: k = 15

9 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 5 cm from A

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{5}{10}$, so θ = 60° at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 5 cm, height 10 sin 60° = $5\sqrt{3}$ cm, area = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 10^2 = \frac{25}{3}\pi$

Area = $\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi$ cm^2

Answer: $\left(\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi\right)$ cm^2

Day 172 **S Tier 10-a-day: Model Solutions**

10 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = x^2 + 2x + 2$.
Find the value of k.

Equate: $x^2 + 2x + 2 = -2x + k$

Rearrange: $x^2 + 4x + 2 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $16 - 4 \times 1 \times (2 - k) = 0$

$4k + 8 = 0$

$k = -2$

Answer: $k = -2$