Day 171 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x − 15)°
Angle PQR = (2x + 25)°
Angle PRS = (5x + 33)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
5x + 33 = 6x + 10
x = 23
(b) Angle PRS = 148° so angle PRQ = 180° − 148° = 32°
Answer: (a) x = 23 (b) 32°

2 Pressure

A concrete slab in the shape of a cuboid weighs 330 N.
Its smallest face has area 250 cm^2. Its largest face has area 800 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 250 cm^2 = 0.025 m^2 and 800 cm^2 = 0.08 m^2
(a) Greatest pressure is on the smallest face: 330 ÷ 0.025 = 13,200 N/m^2
(b) Least pressure is on the largest face: 330 ÷ 0.08 = 4,125 N/m^2
Answer: (a) 13,200 N/m^2 (b) 4,125 N/m^2

3 Tests of divisibility

The 4-digit number 4k77 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 4 + 7 + 7 + k = 18 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Answer: k = 0 or 3 or 6 or 9

4 Recasting solids

A solid cylinder of radius 6 cm and height 6 cm is melted down.
The metal is used to make identical solid spheres of radius 3 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 6^2 \times 6 = 216\pi$
Volume of one sphere = $\frac{4}{3} \times \pi \times 3^3 = 36\pi$
Number of spheres = 216 ÷ 36 = 6
Answer: 6

5 Simple and compound interest

Nia invests £15,000 for 5 years at 3% per year simple interest.
Evan invests £15,000 for 5 years at 3% per year compound interest.
Work out how much more interest Evan earns than Nia.
Give your answer to the nearest penny. (Calculator)

Simple interest: £15,000 × 3% × 5 = £2,250
Compound: £15,000 × 1.03^5 = £17,389.11
Compound interest = £17,389.11 − £15,000 = £2,389.11
Difference = £2,389.11 − £2,250 = £139.11
Answer: £139.11

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6 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + x - 20}{x + 3} \div \frac{x - 4}{x^2 + x - 6}$

Factorise: $\frac{(x + 5)(x - 4)}{(x + 3)} \div \frac{(x - 4)}{(x + 3)(x - 2)}$

Turn the second fraction upside down and multiply: $\frac{(x + 5)(x - 4)}{(x + 3)} \times \frac{(x + 3)(x - 2)}{(x - 4)}$

Cancel $(x - 4)$ and $(x + 3)$

$= (x - 2)(x + 5)$

Answer: $(x - 2)(x + 5)$

7 Estimating the median from grouped data

The table gives information about 78 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 20, 33, 41, 55, 78

The median is the value at position $78 \div 2 = 39$

It is in the class 40 to 50, with 33 values before it, so we need 6 more of the 8 values in this class

Median $\approx 40 + \frac{6}{8} \times 10 = 47.5$

Answer: 47.5

8 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -3$ and $x = 4$, so $y = a(x + 3)(x - 4)$

When $x = 0$, $y = 12$: $a \times 3 \times (-4) = 12$, so $a = -1$

$y = -(x + 3)(x - 4)$

$y = -x^2 + x + 12$

Answer: $y = -x^2 + x + 12$

9 Combined transformations

The point P(4, −2) is reflected in the line $y = x$.
The image is then reflected in the x-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−2, 4)

After the second transformation: P'' = (−2, −4)

(b) Follow a general point (x, y) through both transformations: (x, y) → (y, x) → (y, −x)

The single mapping is (x, y) → (y, −x)

This is a rotation 90° clockwise about the origin

Answer: (a) (−2, −4) (b) Rotation 90° clockwise about the origin

10 Tangency conditions

The line $y = -x + k$ is a tangent to the curve $y = 3x^2 + 4x + 3$.
Find the value of k.

Equate: $3x^2 + 4x + 3 = -x + k$

Rearrange: $3x^2 + 5x + 3 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $25 - 4 \times 3 \times (3 - k) = 0$

$12k - 11 = 0$

$k = \frac{11}{12}$

Answer: $k = \frac{11}{12}$