Day 170 **S Tier 10-a-day: Model Solutions**

1 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 5 cm and inner radius 3.5 cm. (Calculator)
The density of the metal is 7.3 g/cm^3.
The mass of the pipe is 23.4 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 23400 g, so volume = mass ÷ density = 23400 ÷ 7.3 = 3205.5 cm^3
Cross-sectional area = $\pi \times 5^2 - \pi \times 3.5^2 = 40.055$ cm^2
Length = volume ÷ area = 3205.5 ÷ 40.055 = 80.0 cm (3 s.f.)
Answer: 80.0 cm

2 Averages with algebra

Here are five numbers: $2x - 1$, x, $2x + 6$, $2x + 3$, $x + 3$.
The mean of the five numbers is 7.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(2x - 1) + (x) + (2x + 6) + (2x + 3) + (x + 3) = 7 \times 5 = 35$
$8x + 11 = 35$, so $8x = 24$ and $x = 3$
The numbers are 5, 3, 12, 9, 6
(b) Range = $12 - 3 = 9$
Answer: (a) x = 3 (b) 9

3 Column vectors

The vectors $\begin{pmatrix} k \\ 25 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3k \end{pmatrix}$ are parallel.
k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same
$\frac{k}{3} = \frac{25}{3k}$
$k \times 3k = 25 \times 3$
$3k^2 = 75$
$k^2 = 25$
k = 5 (k is positive)
Answer: k = 5

4 Completing the square

(a) Write $-3 - 4x - x^2$ in the form $a - (x + b)^2$
(b) Write down the maximum value of $-3 - 4x - x^2$ and the value of x for which it occurs.

(a) $-3 - 4x - x^2 = -(x^2 + 4x) - 3$
$x^2 + 4x = (x + 2)^2 - 4$
so $-3 - 4x - x^2 = -(x + 2)^2 + 4 - 3 = 1 - (x + 2)^2$
(b) The square is never negative, so the maximum is 1 when $x = -2$
Answer: (a) $1 - (x + 2)^2$ (b) maximum 1 at $x = -2$

5 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 4, y = 448.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 3.

(a) $y = kx^3$
$448 = k \times 64$, so $k = 448 \div 64 = 7$
$y = 7x^3$
(b) $y = 7 \times 27 = 189$
Answer: (a) $y = 7x^3$ (b) 189

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6 Equation of a circle

A circle has centre C(4, −2) and passes through the point P(6, 7).
Find an equation of the circle.

$r^2 = CP^2 = (6 - 4)^2 + (7 - (-2))^2$
$= 4 + 81 = 85$
$(x - 4)^2 + (y + 2)^2 = 85$
Answer: $(x - 4)^2 + (y + 2)^2 = 85$

7 Powers of standard form

Work out $(1.9 \times 10^{-5})^2$
Give your answer in standard form.

$(1.9 \times 10^{-5})^2 = 1.9^2 \times (10^{-5})^2$
$1.9^2 = 3.61$ and $(10^{-5})^2 = 10^{-10}$
$= 3.61 \times 10^{-10}$
Answer: 3.61×10^{-10}

8 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{1}{8}$.

(a) Find the exact value of cos x.
(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{1}{8})^2 = \frac{63}{64}$

x is obtuse, so cos x is negative: $\cos x = -\frac{3\sqrt{7}}{8}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{1}{8} \div (-\frac{3\sqrt{7}}{8}) = -\frac{\sqrt{7}}{21}$

Answer: (a) $-\frac{3\sqrt{7}}{8}$ (b) $-\frac{\sqrt{7}}{21}$

9 Finding a percentage rate

Hana invests £7,300 in an account paying compound interest.
The interest rate is the same percentage each year.
After 2 years the account holds £14,308.
Work out the annual percentage interest rate.

Multiplier for 2 years = £14,308 ÷ £7,300 = 1.96
Yearly multiplier = the square root of 1.96 = 1.4
40% increase each year
Answer: 40%

10 Tangent to a circle: proof

Show that the line $y = 2x + 20$ is a tangent to the circle $x^2 + y^2 = 80$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = 2x + 20$ into $x^2 + y^2 = 80$: $x^2 + (2x + 20)^2 = 80$
Expand: $5x^2 + 80x + 320 = 0$
$b^2 - 4ac = 80^2 - 4 \times 5 \times 320 = 6400 - 6400 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-80}{2 \times 5} = -8$, and $y = 2 \times (-8) + 20 = 4$. The point is (−8, 4)
Answer: (−8, 4)