Day 17 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

30 deer in a forest are caught, marked and put back.
Later, a second sample of 20 deer is caught. 5 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 20

This should match the proportion marked in the whole population: $\frac{5}{20} = \frac{30}{N}$

N = 30 × 20 ÷ 5 = 120

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 120 (b) The population stays the same between samples

2 Error intervals

The width of a table is 900 cm, correct to 1 significant figure.
Write down the error interval for the width of a table, using w.

The value has been rounded to the nearest 100
Half of this is 50, so go 50 below and 50 above
900 − 50 = 850 and 900 + 50 = 950
850 ≤ w < 950

Answer: 850 ≤ w < 950

3 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (2x + 15)°, angle ACB = (3x − 8)° and angle CAQ = 48°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
3x − 8 = 2x + 15
x = 23
(b) Angle PAB = 61°
Angles on the straight line PAQ: angle BAC = 180 − 61 − 48 = 71°

Answer: (a) x = 23 (b) 71°

4 LCM problems

A red light flashes every 4 minutes.
A blue light flashes every 12 minutes.
A green light flashes every 18 minutes.
They all flash together at 10:00 am.
(a) At what time do they next flash together?

Find the LCM of 4, 12 and 18

$4 = 2^2$, $12 = 2^2 \times 3$, $18 = 2 \times 3^2$

LCM = $2^2 \times 3^2 = 36$

36 minutes after 10:00 am is 10:36 am

Answer: 10:36 am

5 Trigonometric equations

Solve 3 cos x° + 7 = 10 for 0 ≤ x ≤ 360

Rearrange: 3 cos x° = 3, so cos x° = 1
From the graph of y = cos x°, cos x° = 1 when x = 0 or 360

Answer: x = 0, 360

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6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on AB with AN : NB = 2 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{2}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a}) = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

(b) OM = $\frac{2}{5}\mathbf{a}$

MN = ON − OM = $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{2}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ (b) $\frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

7 Growth and decay

A culture has 2500 bacteria.
The number of bacteria increases by 3% each hour.
After how many whole hours will there first be more than 5000 bacteria? (Calculator)

Multiplier each hour = 1.03
After 23 hours: $2500 \times 1.03^{23} = 4934$
After 24 hours: $2500 \times 1.03^{24} = 5082$
So the first time it is more than 5000 is after 24 hours
Answer: 24 hours

8 Inverse functions

f(x) = $\frac{5}{x - 5} + 4$
(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(9)$.

(a) Let $y = \frac{5}{x - 5} + 4$, so $y - 4 = \frac{5}{x - 5}$

Make x the subject: $x - 5 = \frac{5}{y - 4}$

$x = \frac{5}{y - 4} + 5$

$f^{-1}(x) = \frac{5}{x - 4} + 5$

(b) $f^{-1}(9) = \frac{5}{9 - 4} + 5 = 6$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{5}{x - 4} + 5}$ (b) 6

9 Mixtures and percentage profit

A shop mixes two types of flour.
Type A costs £13.00 per kg and type B costs £7.00 per kg.
They are mixed in the ratio A : B = 1 : 4.
The mixture is sold for £12.30 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 1 × £13.00 + 4 × £7.00 = £41.00
Cost per kg = £41.00 ÷ 5 = £8.20
Profit per kg = £12.30 − £8.20 = £4.10
Percentage profit = 4.10 ÷ 8.20 × 100 = 50%
Answer: 50%

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on OB with ON : NB = 3 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{3}\mathbf{a} = \frac{2}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{4}t\mathbf{b}$

Equate: $\frac{2}{3}s = 1 - t$ and $1 - s = \frac{3}{4}t$

Solving: $s = \frac{1}{2}$ and $t = \frac{2}{3}$

OX = $\frac{1}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$