Day 169 **S Tier 10-a-day: Model Solutions**

1 Arithmetic sequences

The 5th term of an arithmetic sequence is 30.
The 12th term of the sequence is 51.
(a) Find an expression for the nth term of the sequence.
(b) Is 62 a term of the sequence? You must show your working.

(a) Common difference = (51 − 30) ÷ (12 − 5) = 3
nth term = 3n + c. For n = 5: 3 × 5 + c = 30, so c = 15
nth term = 3n + 15

(b) Solve 3n + 15 = 62: 3n = 47, so n = $\frac{47}{3}$

n is not a whole number, so 62 is not a term
Answer: (a) 3n + 15 (b) No

2 Rationalising the denominator

Rationalise the denominator of $\frac{29}{2\sqrt{3}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{3}$:
$\frac{29}{2\sqrt{3}} = \frac{29\sqrt{3}}{6}$

Answer: $\frac{29\sqrt{3}}{6}$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 34°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 34 = 68°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 68) ÷ 2 = 56°
Answer: (a) 68° (b) 56°

4 Pressure

A wooden block in the shape of a cuboid weighs 150 N.
Its smallest face has area 1500 cm^2. Its largest face has area 3,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1500 cm^2 = 0.15 m^2 and 3,000 cm^2 = 0.3 m^2
(a) Greatest pressure is on the smallest face: 150 ÷ 0.15 = 1,000 N/m^2
(b) Least pressure is on the largest face: 150 ÷ 0.3 = 500 N/m^2
Answer: (a) 1,000 N/m^2 (b) 500 N/m^2

5 Powers of standard form

Work out $(1.1 \times 10^3)^2$
Give your answer in standard form.

$(1.1 \times 10^3)^2 = 1.1^2 \times (10^3)^2$
$1.1^2 = 1.21$ and $(10^3)^2 = 10^6$
$= 1.21 \times 10^6$
Answer: 1.21×10^6

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6 Area and quadratic equations

The diagram shows a rectangle of length $(2x - 1)$ cm, width $(x - 3)$ cm and area 18 cm^2.
(a) Show that $2x^2 - 7x - 15 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(2x - 1)(x - 3) = 18$
Expand: $2x^2 - 7x + 3 = 18$, so $2x^2 - 7x - 15 = 0$
(b) Factorise: $(x - 5)(2x + 3) = 0$
$x = 5$ ($x = -\frac{3}{2}$ is not possible as a length would be negative)
Length = 9 cm and width = 2 cm
Perimeter = $2 \times (9 + 2) = 22$ cm
Answer: (b) 22 cm

7 Rearranging when the subject appears twice

Make u the subject of $v(u + 4) = u - v$

Expand the bracket: $vu + 4v = u - v$
Collect the u terms: $vu - u = -4v - v$
Factorise: $u(v - 1) = -5v$
$u = \frac{5v}{1 - v}$
Answer: $u = \frac{5v}{1 - v}$

8 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 17, 12, 10, 6
Mid-points: 5, 15, 30, 50
$\Sigma fm = 17\times5 + 12\times15 + 10\times30 + 6\times50 = 865$, $\Sigma f = 45$
Mean = $865 \div 45 = 19.2$
Answer: 19.2

9 Loci and regions

ABCD is a rectangle with AB = 11 cm and AD = 11 cm.
A point P inside the rectangle is such that AP $\leq$ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP $\leq$ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2
Answer: 8π cm^2

10 Finding the rate of interest

Rhys invested £20,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £22,472.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $20000 \times m^2 = 22472.00$
$m^2 = 22472.00 \div 20000 = 1.1236$
$m = \sqrt{1.1236} = 1.06$
Rate = $(1.06 - 1) \times 100 = 6\%$
Answer: 6% per year