**Day 168** **S Tier 10-a-day: Model Solutions**

## 1 Combined transformations

The point P(5, 2) is rotated 180° about the origin.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−5, −2)
After the second transformation: P'' = (−2, 5)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (−y, x)
The single mapping is (x, y) → (−y, x)
This is a rotation 90° anticlockwise about the origin
**Answer: (a) (−2, 5) (b) Rotation 90° anticlockwise about the origin**

## 2 Distance-time graphs

The graph shows Maya's cycle ride to a café and back.
(a) Work out Maya's speed on the way back.
(b) Work out Maya's average speed for the whole ride, including the stop.

(a) Return: 15 km in 1 hours, so speed = 15 ÷ 1 = 15 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 3 hours
Average speed = 30 ÷ 3 = 10 km/h
**Answer: (a) 15 km/h (b) 10 km/h**

## 3 Error intervals

The distance to a village is 82 km, correct to the nearest whole number.
Write down the error interval for the distance to a village, using d.

The value has been rounded to the nearest 1
Half of this is 0.5, so go 0.5 below and 0.5 above
82 − 0.5 = 81.5 and 82 + 0.5 = 82.5
$81.5 \leq d < 82.5$
**Answer: $81.5 \leq d < 82.5$**

## 4 Similar shapes

Two shapes are mathematically similar.
The smaller has height 8 cm and area 20 $cm^2$.
The larger has area 125 $cm^2$.
Work out the height of the larger shape.

Ratio of areas = 125 : 20 = 25 : 4
Ratio of lengths = 5 : 2
Height = $8 \times \frac{5}{2} = 20$ cm
**Answer: 20 cm**

## 5 Quadratic inequalities

Solve $x^2 + 3x - 10 \geq 0$

Solve $x^2 + 3x - 10 = 0$: $(x + 5)(x - 2) = 0$, so $x = -5$ or $x = 2$
The graph is a U shape crossing the x-axis at −5 and 2; it is above the axis outside the roots
$x \leq -5$ or $x \geq 2$
**Answer: $x \leq -5$ or $x \geq 2$**

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## 6 Area of a segment

The circle has centre O and radius 10 cm. A and B are points on the circle and angle AOB = 120°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{120}{360} \times \pi \times 10^2 = 104.72$ cm$^2$

Area of triangle OAB = $\frac{1}{2} \times 10 \times 10 \times \sin 120° = 43.301$ cm$^2$

Segment = sector − triangle = 104.72 − 43.301 = 61.418

= 61.4 cm$^2$ (3 s.f.)

**Answer: 61.4 cm$^2$**

## 7 Geometric sequences

The 3rd term of a geometric sequence is 96.
The 6th term of the sequence is −768.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = -768 \div 96 = -8$

$r = \sqrt[3]{-8} = -2$

(b) The 3rd term is $a \times r^2$: $a \times 4 = 96$

$a = 96 \div 4 = 24$

**Answer: (a) r = −2 (b) 24**

## 8 Histograms with an unmarked scale

The histogram shows the masses of parcels delivered by a courier.
280 of them had a mass between 0 and 10 kg.
(a) How many had a mass between 10 and 20 kg?
(b) How many parcels are there in total?

Frequency is proportional to the area of a bar
0 to 10: bar is 7 squares high and 10 wide, area 70, which represents 280
So 1 unit of area represents 280 ÷ 70 = 4
(a) 10 to 20: 4 × 10 = 40 units of area, so 40 × 4 = 160
(b) Total area = 70 + 40 + 140 + 40 + 70 = 360, so total = 360 × 4 = 1440

**Answer: (a) 160 (b) 1440**

## 9 Bounds of speed, density and pressure

A metal block has a mass of 1000 g, correct to the nearest 100.
Its volume is 3.6 cm$^3$, correct to 1 decimal place.
Work out the lower bound of the density.
Give your answer correct to 3 significant figures. (Calculator)

950 ≤ mass < 1050; 3.55 ≤ volume < 3.65
Lower bound of density = smallest mass ÷ biggest volume = 950 ÷ 3.65
= 260.274... = 260 (3 s.f.)

**Answer: 260 g/cm$^3$**

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## 10 Tangency conditions

The line $y = 2x + k$ is a tangent to the curve $y = x^2 - 5x + 1$.
Find the value of k.

Equate: $x^2 - 5x + 1 = 2x + k$

Rearrange: $x^2 - 7x + 1 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $49 - 4 \times 1 \times (1 - k) = 0$

$4k + 45 = 0$

$k = -\frac{45}{4}$

**Answer: $k = -\frac{45}{4}$**