Day 167 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The distance to a village is 2.9 km, truncated to 1 decimal place.
Write down the error interval for the distance to a village, using d.

Truncating cuts off the later digits without rounding.
The smallest possible value is 2.9 and the value must be less than 3
$2.9 \leq d < 3$
Answer: $2.9 \leq d < 3$

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 41°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 41°
Angle COB is an exterior angle of triangle OAC: 41 + 41 = 82°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 82°

3 Proof with indices

Simplify fully $\frac{2^{n+3} - 2^n}{7}$

$2^{n+3} - 2^n = 2^n(2^3 - 1) = 2^n \times 7$

Divide by 7: $\frac{7 \times 2^n}{7} = 2^n$

Answer: 2^n

4 Equations with indices

Solve $3^{x+3} = \frac{1}{9}$

Write the right-hand side as a power of 3: $\frac{1}{9} = 3^{-2}$

Equate the powers: $x + 3 = -2$
$x = -5$
Answer: $x = -5$

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50°$
$x^2 = 61.154$
$x = 7.8$ cm (1 d.p.)
(b) Area = $\frac{1}{2} \times 8 \times 10 \times \sin 50° = 30.6$ cm^2 (3 s.f.)

Answer: (a) 7.8 cm (b) 30.6 cm^2

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6 Upper and lower bounds

a = 0.76 and b = 0.41,
both correct to 2 decimal places.
Work out the lower bound of a ÷ b.
Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.005 above or below
$0.755 \leq a < 0.765$; $0.405 \leq b < 0.415$
Smallest answer: smallest a ÷ biggest b = 0.755 ÷ 0.415
= 1.81928... = 1.82 to 3 significant figures
Answer: 1.82

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 2n + 5$.
(a) Work out the 12th term.
(b) Is 68 a term of the sequence? You must show your working.

(a) $12^2 + 2 \times 12 + 5 = 173$
(b) Solve $n^2 + 2n + 5 = 68$, so $n^2 + 2n - 63 = 0$
Factorise or use the formula: n = 7 or n = −9
n = 7 is a positive whole number, so 68 is term number 7
Answer: (a) 173 (b) Yes, it is the 7th term

8 Reverse percentages

The price of a tablet is increased by 10%.
Later the new price is reduced by 30% in a sale.
The sale price is £369.60.
Work out the price before the increase. (Calculator)

Multipliers: 1.1 and 0.7
Overall multiplier = 1.1 × 0.7 = 0.77
Original × 0.77 = £369.60
Original = £369.60 ÷ 0.77 = £480
Answer: £480

9 Functions that are their own inverse

$f(x) = \frac{-x - 6}{2x + 1}$ where $x \neq -\frac{1}{2}$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x + 1)
$ff(x) = \frac{(-1)(-x - 6) + (-6)(2x + 1)}{2(-x - 6) - (-1)(2x + 1)}$
Top = $x + 6 - 12x - 6 = -11x$
Bottom = $-2x - 12 + 2x + 1 = -11$
$ff(x) = \frac{-11x}{-11} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-x - 6}{2x + 1}$

Answer: (b) $f^{-1}(x) = \frac{-x - 6}{2x + 1}$

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10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 44.4.
Work out the value of x.

Mid-points: 10, 30, 50, 70, 90
Estimated mean = $\Sigma fm \div \Sigma f$, so $(2790 + 30x) \div (57 + x) = 44.4$
$2790 + 30x = 44.4(57 + x) = 2530.8 + 44.4x$
$(30 - 44.4)x = 2530.8 - 2790$
$-14.4x = -259.2$, so $x = 18$
Answer: x = 18