Day 166 **S Tier 10-a-day: Model Solutions**

1 Bounds of speed, density and pressure

A runner covers a distance of 60 m, correct to 1 significant figure.
The race takes 8.4 s, correct to 2 significant figures.
Work out the lower bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

$55 \leq$ distance < 65; $8.35 \leq$ time < 8.45
Lower bound of speed = smallest distance ÷ biggest time = 55 ÷ 8.45
= 6.50888... = 6.51 (3 s.f.)
Answer: 6.51 m/s

2 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 9 : 8.
Then $\frac{2}{3}$ of the blue and $\frac{1}{2}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{1}{3} \times 9 = 3$ of the blue part, $\frac{1}{2} \times 8 = 4$ of the green part
3 : 4
Multiply by 1: 3 : 4
Answer: 3 : 4

3 Recasting solids

A solid metal sphere of radius 6 cm is melted down and recast into a solid cylinder of radius 12 cm.
Work out the height of the cylinder.
The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 6^3 = 288\pi$
Volume of cylinder = $\pi \times 12^2 \times h = 144\pi h$
$144h = 288$
$h = 2$ cm
Answer: 2 cm

4 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 15 ÷ 5 = 3 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 15 + 15 \times 20 + \frac{1}{2} \times 15 \times 15 = \frac{75}{2} + 300 + \frac{225}{2}$
= 450
Answer: (a) 3 m/s^2 (b) 450 m

Day 166 **S Tier 10-a-day: Model Solutions**

5 Winning a match

Leila and Tariq play a match. The first to win 2 table tennis games wins the match.

The probability that Leila wins any one of the table tennis games is $\frac{3}{5}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Leila wins then loses) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(Leila loses then wins) = $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

Total = $2 \times \frac{6}{25} = \frac{12}{25}$

Answer: $\frac{12}{25}$

6 Prime factors and indices

Write $36^3 \div 6^3$ as a product of powers of prime numbers.

$36 = 2^2 \times 3^2$, $6 = 2 \times 3$

$36^3 = 2^6 \times 3^6$

$6^3 = 2^3 \times 3^3$

Subtract the powers of each prime: $2^3 \times 3^3$

Answer: $2^3 \times 3^3$

7 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.

(a) Work out the height VM of the pyramid.

(b) Work out the volume of the pyramid.

The diagram is not drawn accurately.

(a) $AC^2 = 12^2 + 12^2 = 288$, so $AM^2 = (AC \div 2)^2 = 72$

$VM^2 = VA^2 - AM^2 = 11^2 - 72 = 49$

VM = 7 cm

(b) Volume = $\frac{1}{3} \times 12^2 \times 7 = 336$ cm^3

Answer: (a) 7 cm (b) 336 cm^3

8 Estimating the mean from a histogram table

The table gives information about the times of some people.

Work out an estimate for the mean time.

Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 11, 38, 30, 12

Mid-points: 5, 20, 45, 80

$\Sigma fm = 11 \times 5 + 38 \times 20 + 30 \times 45 + 12 \times 80 = 3125$, $\Sigma f = 91$

Mean = $3125 \div 91 = 34.3$

Answer: 34.3

9 Counting factors

How many factors does 13,500 have?

Include 1 and 13,500.

$13{,}500 = 2^2 \times 3^3 \times 5^3$

Add 1 to each power and multiply: $3 \times 4 \times 4 = 48$

Answer: 48

Day 166 **S Tier 10-a-day: Model Solutions**

10 Equal roots and real roots

The equation $x^2 + kx + 4 = 0$ has exactly one solution.
Find the possible values of k.
Give any surds in their simplest form.

One solution means $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 4 = 0$, so $k^2 = 16$
$k = \pm\sqrt{16} = \pm 4$
Answer: k = ±4