Day 165 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 23 is taken. Work out how many come from Year 10.
(b) A different stratified sample has 15 from Year 10. Work out the size of this sample.

Total = 150 + 100 + 200 + 125 = 575

(a) Year 10: $\frac{125}{575} \times 23 = 5$

(b) Fraction sampled = $15 \div 125 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 575 = 69$

Answer: (a) 5 (b) 69

2 Angles in polygons

The sum of the interior angles of a polygon is 1980°.
Work out the number of sides of the polygon.

Sum of interior angles = $(n - 2) \times 180°$
$(n - 2) \times 180 = 1980$
$n - 2 = 1980 \div 180 = 11$
$n = 13$
Answer: 13

3 Quadratic sequences

Here are the first five terms of a quadratic sequence.
$-3, 5, 21, 45, 77$
Find an expression for the nth term.

First differences: 8, 16, 24, 32
Second differences: 8, 8, 8, so $a = 8 \div 2 = 4$
Subtract $4n^2$ from each term: $-7, -11, -15, -19, -23$
The remaining linear sequence has nth term $-4n - 3$
nth term = $4n^2 - 4n - 3$
Answer: $4n^2 - 4n - 3$

4 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 80 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a time greater than 45.

(a) Median at cumulative frequency 40, so median = 40
Lower quartile at 20, so LQ = 30. Upper quartile at 60, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 45, cumulative frequency = 50
Number greater = 80 − 50 = 30
Answer: (a) median 40, IQR 20 (b) 30

5 Quadratic formula and surds

Solve $x^2 + 8x - 8 = 0$
Give your answers in the form $a \pm b\sqrt{c}$

Complete the square: $(x + 4)^2 - 16 - 8 = 0$
$(x + 4)^2 = 24$
$x + 4 = \pm\sqrt{24} = \pm 2\sqrt{6}$
$x = -4 \pm 2\sqrt{6}$
Answer: $x = -4 \pm 2\sqrt{6}$

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6 Using a line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The equation of the line of best fit is y = 4x + 8.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = 4 \times 7 + 8 = 36$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 36 (b) x = 15 is outside the range of the data

7 LCM problems

A church bell rings every 6 minutes.
Another bell rings every 8 minutes.
A third bell rings every 24 minutes.
They all ring together at 7:00 am.
How many times do they all ring together between 7:00 am and 3:00 pm, including both 7:00 am and 3:00 pm if they do?

Find the LCM of 6, 8 and 24
$6 = 2 \times 3$, $8 = 2^3$, $24 = 2^3 \times 3$
LCM $= 2^3 \times 3 = 24$
Total time from 7:00 am to 3:00 pm = 480 minutes
480 ÷ 24 = 20, so there are 20 more times after the start
Including the start: 20 + 1 = 21
Answer: 21

8 Transformations of graphs

The graph of y = f(x) has a maximum point at (5, 5).
Write down the coordinates of the turning point of the graph of
(a) y = f(x − 1)
(b) y = f(x + 2) − 2

(a) Start with the turning point (5, 5)
Apply the transformation to the point: (6, 5)
(b) f(x + 2) changes the x-coordinate to 3, and f(x) − 2 changes the y-coordinate to 3
Turning point: (3, 3)
Answer: (a) (6, 5) (b) (3, 3)

9 Surds and fractions

Simplify $\frac{1}{4+\sqrt{6}} - \frac{1}{4-\sqrt{6}}$
Give your answer with a rational denominator.

Common denominator: $(4+\sqrt{6})(4-\sqrt{6}) = 16 - 6 = 10$
Numerator: $(4-\sqrt{6}) - (4+\sqrt{6})$
$= -2\sqrt{6}$
$\frac{-2\sqrt{6}}{10} = -\frac{\sqrt{6}}{5}$
Answer: $\mathbf{-\frac{\sqrt{6}}{5}}$

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10 Loci and regions

ABCD is a rectangle with AB = 13 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2

Answer: $\frac{25}{2}\pi$ cm^2