Day 164 **S Tier 10-a-day: Model Solutions**

1 Fractions

Work out $(4\frac{2}{3} - 2\frac{1}{3}) \times \frac{2}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{14}{3} - \frac{7}{3} = \frac{7}{3}$

$\frac{7}{3} \times \frac{2}{3} = \frac{14}{9}$

$= 1\frac{5}{9}$

Answer: $1\frac{5}{9}$

2 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 2 : 1.
70% of the Year 10 students and 40% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 60 students in total: 40 Year 10 students and 20 Year 11 students
70% of 40 = 28 and 40% of 20 = 8
Together: 36 out of 60
36 ÷ 60 × 100 = 60%
Answer: 60%

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 54°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 54°
Angle COB is an exterior angle of triangle OAC: 54 + 54 = 108°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 108°

4 Geometric sequences

The 3rd term of a geometric sequence is 384.
The 5th term of the sequence is 1536.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 1536 \div 384 = 4$
$r = \sqrt{4} = 2$ (positive, since all terms are positive)
(b) The 3rd term is $a \times r^2$: $a \times 4 = 384$
$a = 384 \div 4 = 96$
Answer: (a) r = 2 (b) 96

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5 Simple and compound interest

Jess invests £1,000 for 3 years at 1.5% per year simple interest.
Tom invests £1,000 for 3 years at 1.5% per year compound interest.
Work out how much more interest Tom earns than Jess.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 1.5% × 3 = £45
Compound: £1,000 × 1.015^3 = £1,045.68
Compound interest = £1,045.68 − £1,000 = £45.68
Difference = £45.68 − £45 = £0.68
Answer: £0.68

6 Tangent to a circle: proof

Show that the line $y = 3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = 3x - 10$ into $x^2 + y^2 = 10$: $x^2 + (3x - 10)^2 = 10$
Expand: $10x^2 - 60x + 90 = 0$
$b^2 - 4ac = (-60)^2 - 4 \times 10 \times 90 = 3600 - 3600 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-(-60)}{2 \times 10} = 3$, and $y = 3 \times 3 - 10 = -1$. The point is $(3, -1)$
Answer: (3, −1)

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mass greater than 55.

(a) Median at cumulative frequency 40, so median = 50
Lower quartile at 20, so LQ = 40. Upper quartile at 60, so UQ = 60
Interquartile range = 60 − 40 = 20
(b) At 55, cumulative frequency = 50
Number greater = 80 − 50 = 30
Answer: (a) median 50, IQR 20 (b) 30

8 Surds with unknowns

Given that $(a + \sqrt{15})(b + \sqrt{15}) = 30 + 8\sqrt{15}$,
where a and b are positive integers with $a < b$, find the values of a and b.

$(a + \sqrt{15})(b + \sqrt{15}) = ab + (a + b)\sqrt{15} + 15$
Compare the $\sqrt{15}$ terms: $a + b = 8$
Compare the integer terms: $ab + 15 = 30$, so $ab = 15$
Two numbers with sum 8 and product 15 are 3 and 5
Answer: a = 3, b = 5

9 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 6 : 5.
Then 10 apples are removed and 3 pears are removed.
The ratio is now 2 : 3.
How many pears were there at the start?

Let the numbers at the start be 6k and 5k
$(6k - 10) : (5k - 3) = 2 : 3$, so $3(6k - 10) = 2(5k - 3)$
$18k - 30 = 10k - 6$
$8k = 24$, so $k = 3$
Number of pears at the start: $5 \times 3 = 15$
Answer: 15

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on OB with ON : NB = 2 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{2}{5}t\mathbf{b}$

Equate: $\frac{1}{5}s = 1 - t$ and $1 - s = \frac{2}{5}t$

Solving: $s = \frac{15}{23}$ and $t = \frac{20}{23}$

OX = $\frac{3}{23}\mathbf{a} + \frac{8}{23}\mathbf{b}$

Answer: (a) $\frac{1}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{23}\mathbf{a} + \frac{8}{23}\mathbf{b}$