Day 163 **S Tier 10-a-day: Model Solutions**

1 Algebraic fractions: adding and subtracting

Write $\frac{4}{2x-3} + \frac{x-1}{x+1}$ as a single fraction.

Simplify the numerator.

The common denominator is $(2x - 3)(x + 1)$

$$\frac{4}{2x-3} + \frac{x-1}{x+1} = \frac{4(x+1) + (x-1)(2x-3)}{(2x-3)(x+1)}$$

Expand the numerator: $2x^2 - x + 7$

$$\frac{2x^2 - x + 7}{(2x-3)(x+1)}$$

Answer: $\mathbf{\frac{2x^2 - x + 7}{(2x-3)(x+1)}}$

2 Mean of a changing set

The mean of 9 numbers is 12.
A new number is added and the mean of all 10 numbers is 15.7.
Work out the new number.

Total of the 9 numbers = $9 \times 12 = 108$
New total = $10 \times 15.7 = 157$
New number = $157 - 108 = 49$
Answer: 49

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 42°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 42°
Angle AOB = $180 - 2 \times 42 = 96°$
(b) The angle at the circumference is half the angle at the centre: $96 \div 2 = 48°$
Answer: (a) 96° (b) 48°

4 Fraction problems

A tank is $\frac{1}{3}$ full.

16 more litres of water are added and the tank is then $\frac{3}{5}$ full.

Work out the capacity of the tank.

Fraction added = $\frac{3}{5} - \frac{1}{3} = \frac{4}{15}$

$\frac{4}{15}$ of the capacity is 16 litres

Capacity = $16 \div \frac{4}{15} = 60$

Answer: 60 litres

5 Radius from chord and height of arc

A circular arc has a chord AB of length 10 cm.
The greatest distance between the chord and the arc is 1 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 1$
AM = $10 \div 2 = 5$. Pythagoras in triangle OMA: $r^2 = (r - 1)^2 + 5^2$
$r^2 = r^2 - 2r + 1 + 25$
$2r = 26$, so $r = 13$ cm
Answer: 13 cm

Day 163 **S Tier 10-a-day: Model Solutions**

6 Rationalising the denominator

Write $\frac{5}{3-\sqrt{6}}$ in the form $\frac{p+q\sqrt{6}}{r}$
where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(3+\sqrt{6})$:

$\frac{5}{3-\sqrt{6}} = \frac{5(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})}$

Denominator: $9 - 6 = 3$

$= \frac{15+5\sqrt{6}}{3}$

Answer: $\mathbf{\frac{15+5\sqrt{6}}{3}}$

7 Reverse percentage growth and decay

The value of a house increases by 12% each year.
After 3 years it is worth £52,684.80.
Work out its value 3 years ago. (Calculator)

Yearly multiplier = 1.12

Original $\times 1.12^3$ = £52,684.80

Original = £52,684.80 $\div 1.12^3$ = £37,500

Answer: £37,500

8 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 120 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mass greater than 25.

(a) Median at cumulative frequency 60, so median = 40
Lower quartile at 30, so LQ = 30. Upper quartile at 90, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 25, cumulative frequency = 24
Number greater = 120 − 24 = 96
Answer: (a) median 40, IQR 20 (b) 96

9 Tangency conditions

The line $y = x + k$ is a tangent to the circle $x^2 + y^2 = 30$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = x + k$: $x^2 + (x + k)^2 = 30$

Expand: $2x^2 + 2kx + k^2 - 30 = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $(2k)^2 - 4 \times 2 \times (k^2 - 30) = 0$

$-4k^2 + 240 = 0$, so $k^2 = 60$

$k = \pm 2\sqrt{15}$

Answer: k $= \pm 2\sqrt{15}$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on OB with ON : NB = 2 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{3}\mathbf{a} = \frac{2}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{2}{5}t\mathbf{b}$

Equate: $\frac{2}{3}s = 1 - t$ and $1 - s = \frac{2}{5}t$

Solving: $s = \frac{9}{11}$ and $t = \frac{5}{11}$

OX = $\frac{6}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{6}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$