Day 162 **S Tier 10-a-day: Model Solutions**

1 Expanding three brackets

Expand and simplify $(x - 1)^2(x - 3)$

Expand the first two brackets: $(x - 1)^2 = x^2 - 2x + 1$

Multiply by $(x - 3)$: $x(x^2 - 2x + 1) - 3(x^2 - 2x + 1)$

$= x^3 - 5x^2 + 7x - 3$

Answer: $x^3 - 5x^2 + 7x - 3$

2 Distance-time graphs

The graph shows Kira's cycle ride to a café and back.

(a) Work out Kira's speed on the way back.

(b) Work out Kira's average speed for the whole ride, including the stop.

(a) Return: 10 km in 0.5 hours, so speed = 10 ÷ 0.5 = 20 km/h

(b) Total distance = 2 × 10 = 20 km, total time = 2.5 hours

Average speed = 20 ÷ 2.5 = 8 km/h

Answer: (a) 20 km/h (b) 8 km/h

3 Trigonometric graphs

The diagram shows the graph of $y = a \cos bx$, where a and b are positive integers.

(a) Write down the value of a and the value of b.

(b) Work out the value of y when $x = 120°$.

(a) The maximum value is 4 and the minimum is -4, so $a = 4$

The graph completes 4 full cycles between 0° and 360°, so $b = 4$

(b) $y = 4 \cos (4 \times 120°) = 4 \cos 480°$

$\cos 120° = -\frac{1}{2}$, so $y = -2$

Answer: (a) $a = 4$, $b = 4$ (b) $y = -2$

4 Simplifying surds

Write $\sqrt{175} - \sqrt{112} + \sqrt{7}$ in the form $a\sqrt{7}$ where a is an integer.

$\sqrt{175} = 5\sqrt{7}$; $\sqrt{112} = 4\sqrt{7}$

$(5 - 4 + 1)\sqrt{7} = 2\sqrt{7}$

Answer: $2\sqrt{7}$

5 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 2x - 24}{x^2 - 13x + 42}$

Factorise the top: $(x - 6)(x + 4)$

Factorise the bottom: $(x - 6)(x - 7)$

Cancel the common factor $(x - 6)$: $\frac{x + 4}{x - 7}$

Answer: $\frac{x + 4}{x - 7}$

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6 Fraction problems

A barrel is $\frac{3}{4}$ full.

33 more litres of oil are added and the barrel is then $\frac{9}{10}$ full.

Work out the capacity of the barrel.

Fraction added = $\frac{9}{10} - \frac{3}{4} = \frac{3}{20}$

$\frac{3}{20}$ of the capacity is 33 litres

Capacity = $33 \div \frac{3}{20} = 220$

Answer: 220 litres

7 Inverse proportion

P is inversely proportional to the square root of t.
When t = 4, P = 3.
Find the positive value of t when P = 6.

$P = \frac{k}{\sqrt{t}}$

$3 = \frac{k}{2}$, so $k = 3 \times 2 = 6$

$6 = \frac{6}{\sqrt{t}}$, so $\sqrt{t} = \frac{6}{6} = 1$

$t = 1^2 = 1$

Answer: t = 1

8 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on AB with AN : NB = 3 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{3}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a}) = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(b) OM = $\frac{1}{4}\mathbf{a}$

MN = ON − OM = $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} - \frac{1}{4}\mathbf{a} = \frac{3}{4}\mathbf{b}$

MN = $\frac{3}{4}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$ (b) MN = $\frac{3}{4}\mathbf{b}$, parallel to OB

9 Tests of divisibility

The 5-digit number 2230k is a multiple of 6.
k is a single digit.
Find all the possible values of k.

A multiple of 6 is even and a multiple of 3
The last digit k must be even
Digit sum = 2 + 2 + 3 + 0 + k = 7 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Even values only
Answer: k = 2 or 8

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10 Ratio and money transferred

Dina and Nadia share some money in the ratio 4 : 5.
Dina gives Nadia £26.
The ratio of their money is now 2 : 7.
How much money do they have altogether?

Let the amounts be 4k and 5k, so the total is 9k

Dina now has $4k - 26$, which is $\frac{2}{9}$ of the total

$9(4k - 26) = 2 \times 9k$
$36k - 234 = 18k$, so $18k = 234$ and $k = 13$
Total = 9×13 = £117
Answer: £117