Day 161 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3x - y^3$
when $x = -5$, $y = -2$ and $z = 2$.

Substitute the values: $3 \times (-5) - (-2)^3$
$= -15 - (-8)$
$= -7$
Answer: -7

2 Line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The line of best fit passes through (2, 13) and (8, 16).
(a) Find its equation in the form $y = mx + c$.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{16 - 13}{8 - 2} = \frac{3}{6} = 0.5$
$y = 0.5x + c$: using (2, 13), $13 = 0.5 \times 2 + c$, so $c = 12$
$y = 0.5x + 12$
(b) For each extra degree of temperature, the number of ice creams sold goes up by 0.5
Answer: (a) $y = 0.5x + 12$ (b) For each extra degree of temperature, the number of ice creams sold goes up by 0.5

3 Solving quadratics by factorising

Solve $8x^2 - 14x + 5 = 0$

Factorise: $(4x - 5)(2x - 1) = 0$
$4x - 5 = 0$ or $2x - 1 = 0$
$x = \frac{5}{4}$ or $x = \frac{1}{2}$
Answer: $x = \frac{1}{2}$ or $x = \frac{5}{4}$

4 Standard form problems

A library has 8×10^4 books.
Each book has 2.4×10^4 words.
Work out the total number of words in all the books.
Give your answer in standard form.

Total $= (8 \times 10^4) \times (2.4 \times 10^4)$
$8 \times 2.4 = 19.2$ and $10^4 \times 10^4 = 10^8$
$= 1.92 \times 10^9$
Answer: 1.92×10^9 words

5 Changing the subject

Make u the subject of $v = \frac{2}{\sqrt{u}}$

Multiply by $\sqrt{u}$: $v\sqrt{u} = 2$
Divide by v: $\sqrt{u} = \frac{2}{v}$
Square both sides: $u = \frac{4}{v^2}$
Answer: $u = \frac{4}{v^2}$

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6 Relative speed

Amir sets off at 10:00 and cycles at 24 km/h along a path.
Priya sets off from the same place at 10:15 and cycles at 32 km/h along the same path.
At what time does Priya catch up with Amir?

When Priya starts, Amir is 0.25 × 24 = 6 km ahead
Priya gains 32 − 24 = 8 km every hour
Time to catch up = 6 ÷ 8 = 0.75 hours = 45 minutes
10:15 + 45 minutes = 11:00
Answer: 11:00

7 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 21 cm, top radius 6 cm and base radius 24 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+21} = \frac{6}{24}$

24 h = 6(h + 21), so 18 h = 126 and h = 7

Large cone: $\frac{1}{3} \times \pi \times 24^2 \times 28 = 5376\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 7 = 84\pi$

Frustum = $5376\pi - 84\pi = 5292\pi$ cm^3
Answer: 5292π cm^3

8 HCF and LCM

The HCF of two numbers is 8 and their LCM is 480.
The smaller number is 40.
Work out the larger number.

HCF × LCM = product of the two numbers
8 × 480 = 3840
Larger number = 3840 ÷ 40 = 96
Answer: 96

9 Solving trigonometric equations

Solve $\tan x = \frac{\sqrt{3}}{3}$ for $0° \leq x \leq 360°$.

Do not use a calculator.

The related acute angle satisfies $\tan \theta = \frac{\sqrt{3}}{3}$, so $\theta = 30°$

tangent is positive in the 1st and 3rd quadrants, so x = 30° and x = 180° + 30°
x = 30° and x = 210°
Answer: x = 30° or x = 210°

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10 Percentages with an unknown rate

The price of a car is decreased by x%.
The new price is then increased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$

$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and $x = 50$

Answer: x = 50