Day 160 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

40 deer in a forest are caught, marked and put back.
Later, a second sample of 120 deer is caught. 2 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 2 out of 120

This should match the proportion marked in the whole population: $\frac{2}{120} = \frac{40}{N}$

N = 40 × 120 ÷ 2 = 2400

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 2400 (b) The population stays the same between samples

2 Enlargement

Triangle T has vertices A(6, −2), B(7, 0), C(6, −3).
T is enlarged with scale factor 3 and centre (4, −2).
(a) Write down the coordinates of the image of A.
(b) The area of T is 6 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (6 − 4, −2 − (−2)) = (2, 0)
Multiply by 3: (6, 0)
Image of A = centre + this vector = (10, −2)

(b) Area scale factor = $(3)^2 = 9$
Area = 6 × 9 = 54 square units

Answer: (a) (10, −2) (b) 54

3 Functions that are their own inverse

$f(x) = \frac{6x - 2}{2x - 6}$ where $x \neq 3$

(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x − 6)

$ff(x) = \frac{6(6x - 2) + (-2)(2x - 6)}{2(6x - 2) - 6(2x - 6)}$

Top = 36x − 12 − 4x + 12 = 32x
Bottom = 12x − 4 − 12x + 36 = 32

$ff(x) = \frac{32x}{32} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{6x - 2}{2x - 6}$

Answer: (b) $f^{-1}(x) = \frac{6x - 2}{2x - 6}$

4 Averages with algebra

Here are five numbers: x + 1, 3x − 2, x + 3, 3x + 1, x − 1.
The mean of the five numbers is 13.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = (x + 1) + (3x − 2) + (x + 3) + (3x + 1) + (x − 1) = 13 × 5 = 65
9x + 2 = 65, so 9x = 63 and x = 7
The numbers are 8, 19, 10, 22, 6

(b) Range = 22 − 6 = 16

Answer: (a) x = 7 (b) 16

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5 Bounds in geometry

A runner covers 600 metres, correct to the nearest 10 metres, in a time of 22.6 seconds, correct to 1 decimal place. (Calculator)
Work out the upper bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 595 and 605 metres; time is between 22.55 and 22.65 seconds
Speed = distance ÷ time. For the upper bound use the greatest distance and the smallest time
605 ÷ 22.55 = 26.8 m/s (3 s.f.)
Answer: 26.8 m/s

6 Tree diagrams without replacement

A bag contains 2 red counters, 2 blue counters and 3 green counters.
Two counters are taken at random without replacement.
Work out the probability that (a) both are the same colour, (b) they are different colours.

(a) Add the ways: $\frac{2}{7}\times\frac{1}{6} + \frac{2}{7}\times\frac{1}{6} + \frac{3}{7}\times\frac{2}{6} = \frac{5}{21}$

(b) P(different) = 1 − P(same) = $1 - \frac{5}{21} = \frac{16}{21}$

Answer: (a) $\frac{5}{21}$ (b) $\frac{16}{21}$

7 Laws of indices

Write $\frac{4^{n-2}\times 16^{n-1}}{2^n}$ in the form 2^{an+c}
where a and c are integers.

$4 = 2^2$, $16 = 2^4$, $2 = 2^1$
$4^{n-2} = 2^{2(n-2)}$, $16^{n-1} = 2^{4(n-1)}$, $2^n = 2^n$
Exponent: $2(n-2) + 4(n-1) - n = 5n - 8$
Answer: a = 5, c = −8

8 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 40%.
Lena earns £58,000 in a year.
Work out how much income tax Lena pays.

Taxed at 20%: £30,000
20% of £30,000 = £6,000
Taxed at 40%: £58,000 − £15,000 − £30,000 = £13,000
40% of £13,000 = £5,200
Total tax = £6,000 + £5,200 = £11,200
Answer: £11,200

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9 Cone made from a sector

A sector of a circle has radius 10 cm and angle 72°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{72}{360} \times 2\pi \times 10 = 4\pi$ cm

This is the circumference of the base: $2\pi r = 4\pi$, so r = 2 cm

(b) Slant height = 10 cm, so height$^2 = 10^2 - 2^2 = 96$, height = $4\sqrt{6}$ cm

Volume = $\frac{1}{3} \times \pi \times 2^2 \times 4\sqrt{6} = \frac{16\sqrt{6}}{3}\pi$ cm^3

Answer: (a) 2 cm (b) $\frac{16\sqrt{6}}{3}\pi$ cm^3

10 Simultaneous equations

Solve the simultaneous equations
$4x + 4y = -12$
$-3x + 2y = 4$

To match the x terms: (1) × 3 gives $12x + 12y = -36$, and (2) × 4 gives $-12x + 8y = 16$
Add the equations to eliminate x: $20y = -20$
$y = -1$
Substitute $y = -1$ into (1): $4x - 4 = -12$
$4x = -8$, so $x = -2$
Answer: $x = -2$, $y = -1$