Day 16 **S Tier 10-a-day: Model Solutions**

1 Algebraic proof

Show algebraically that the product of any two consecutive even numbers is a multiple of 8.

Let the even numbers be 2k and 2k + 2, where k is an integer.

$2k(2k + 2) = 4k^2 + 4k = 4k(k + 1)$

k and k + 1 are consecutive integers, so one of them is even, and $k(k + 1) = 2 \times$ an integer

So $4k(k + 1) = 8 \times$ an integer, a multiple of 8

Answer: Proved

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

92%, $\frac{7}{8}$, $\frac{14}{15}$, $\frac{7}{11}$

Convert each to a decimal: 92% = 0.92, $\frac{7}{8}$ = 0.875, $\frac{14}{15}$ = 0.933, $\frac{7}{11}$ = 0.636

In order: $\frac{7}{11}$, $\frac{7}{8}$, 92%, $\frac{14}{15}$

Answer: $\frac{7}{11}$, $\frac{7}{8}$, 92%, $\frac{14}{15}$

3 Inequalities

(a) Solve $-2 - 3x \leq -20$

(b) n is an integer. Write down all the values of n that satisfy

$-2 < 2n - 4 < 12$

(a) Add 2: $-3x \leq -18$

Divide by -3 and reverse the inequality sign: $x \geq 6$

(b) Add 4: $2 < 2n < 16$

Divide by 2: $1 < n < 8$

Integers: 2, 3, 4, 5, 6, 7

Answer: (a) $x \geq 6$ (b) 2, 3, 4, 5, 6, 7

4 Averages with algebra

Here are five numbers: $3x - 1$, $x - 3$, $x - 1$, $3x$, $x + 4$.

The mean of the five numbers is 7.

(a) Work out the value of x.

(b) Work out the range of the five numbers.

(a) Total = $(3x - 1) + (x - 3) + (x - 1) + (3x) + (x + 4) = 7 \times 5 = 35$

$9x - 1 = 35$, so $9x = 36$ and $x = 4$

The numbers are 11, 1, 3, 12, 8

(b) Range = $12 - 1 = 11$

Answer: (a) x = 4 (b) 11

5 Standard form

Work out $(6.6 \times 10^8) + (2.8 \times 10^7)$

Give your answer in standard form.

Write the first number as a multiple of 10^7: $6.6 \times 10^8 = 66 \times 10^7$

$(66 + 2.8) \times 10^7 = 68.8 \times 10^7$

$= 6.88 \times 10^8$

Answer: 6.88×10^8

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6 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.
red: x; blue: 2x + 0.2; green: 3x + 0.2; yellow: 0.3
(a) Work out the value of x.
(b) The biased spinner is used 200 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: $x + 2x + 0.2 + 3x + 0.2 + 0.3 = 1$
$6x + 0.7 = 1$, so $6x = 0.3$
$x = 0.05$
(b) P(red) = x = 0.05, so $200 \times 0.05 = 10$
Answer: (a) x = 0.05 (b) 10

7 Equation of a circle

A circle has centre C(4, 5) and passes through the point P(−2, 1).
Find an equation of the circle.

$r^2 = CP^2 = (-2 - 4)^2 + (1 - 5)^2$
$= 36 + 16 = 52$
$(x - 4)^2 + (y - 5)^2 = 52$
Answer: $(x - 4)^2 + (y - 5)^2 = 52$

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = $-3\mathbf{a} - 3\mathbf{b}$, OQ = $-6\mathbf{a} + 6\mathbf{b}$ and OR = $-7\mathbf{a} + 9\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-3\mathbf{a} + 9\mathbf{b}$
QR = OR − OQ = $-\mathbf{a} + 3\mathbf{b}$
QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 1
Answer: (b) 3 : 1

9 Surds with unknowns

Given that $(a + \sqrt{5})(b + \sqrt{5}) = 11 + 7\sqrt{5}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{5})(b + \sqrt{5}) = ab + (a + b)\sqrt{5} + 5$
Compare the $\sqrt{5}$ terms: $a + b = 7$
Compare the integer terms: $ab + 5 = 11$, so $ab = 6$
Two numbers with sum 7 and product 6 are 1 and 6
Answer: a = 1, b = 6

10 Loci and regions

ABCD is a rectangle with AB = 14 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2

Answer: 8π cm^2