Day 159 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.73, $\frac{3}{8}$, $\frac{7}{12}$, $\frac{1}{5}$

Convert each to a decimal: 0.73 = 0.73, $\frac{3}{8}$ = 0.375, $\frac{7}{12}$ = 0.583, $\frac{1}{5}$ = 0.2

In order: $\frac{1}{5}$, $\frac{3}{8}$, $\frac{7}{12}$, 0.73

Answer: $\frac{1}{5}$, $\frac{3}{8}$, $\frac{7}{12}$, 0.73

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 46°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 46 = 134°
(b) The angle at the circumference is half the angle at the centre: 134 ÷ 2 = 67°
Answer: (a) 134° (b) 67°

3 Iteration

Use the iteration formula $x_{n+1} = \sqrt{2x_n + 7}$ with $x_0 = 1$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{2 \times 1 + 7} = 3.00000...$
$x_2 = \sqrt{2 \times x_1 + 7} = 3.60555...$
$x_3 = \sqrt{2 \times x_2 + 7} = 3.76976...$
Rounded: $x_1 = 3.000$, $x_2 = 3.606$, $x_3 = 3.770$
Answer: $x_1 = 3.000$, $x_2 = 3.606$, $x_3 = 3.770$

4 Standard form

Work out $(6.6 \times 10^4) \div (1.5 \times 10^8)$
Give your answer in standard form.

$6.6 \div 1.5 = 4.4$
$10^4 \div 10^8 = 10^{-4}$
$= 4.4 \times 10^{-4}$
Answer: 4.4×10^{-4}

5 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-8 - 0}{0 - (-3)} = -\frac{8}{3}$

Perpendicular gradient = $-1 \div (-\frac{8}{3}) = \frac{3}{8}$

M passes through B(−3, 0): $0 = \frac{3}{8} \times (-3) + c$, so $c = \frac{9}{8}$

$y = \frac{3}{8}x + \frac{9}{8}$

Answer: $y = \frac{3}{8}x + \frac{9}{8}$

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6 Independent events and probability rules

A and B are two events.
P(A) = 0.7, P(B) = 0.8 and P(A or B) = 0.94
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.94 = 0.7 + 0.8 - P(A \cap B)$, so $P(A \cap B) = 0.56$
(b) $P(A) \times P(B) = 0.7 \times 0.8 = 0.56$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.56 (b) Yes

7 Shaded areas: exact answers

The diagram shows an equilateral triangle inside a circle of radius 4 cm.
The three vertices of the triangle lie on the circle.
Work out the area of the shaded region. Give your answer in the form $a\pi - b\sqrt{3}$.

Split the triangle into 3 isosceles triangles with two sides of 4 cm and an angle of 120° between them

Area of the triangle = $3 \times \frac{1}{2} \times 4 \times 4 \times \sin 120° = 24 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

Area of circle = $\pi \times 4^2 = 16\pi$
Shaded area = $16\pi - 12\sqrt{3}$ cm^2
Answer: $(16\pi - 12\sqrt{3})$ cm^2

8 Reverse percentages

The price of a games console is increased by 40%.
Later the new price is reduced by 20% in a sale.
The sale price is £593.60.
Work out the price before the increase. (Calculator)

Multipliers: 1.4 and 0.8
Overall multiplier = $1.4 \times 0.8 = 1.12$
Original × 1.12 = £593.60
Original = £593.60 ÷ 1.12 = £530
Answer: £530

9 Surds and fractions

Simplify $\frac{1}{5+\sqrt{10}} + \frac{1}{5-\sqrt{10}}$
Give your answer with a rational denominator.

Common denominator: $(5+\sqrt{10})(5-\sqrt{10}) = 25 - 10 = 15$
Numerator: $(5-\sqrt{10}) + (5+\sqrt{10})$
$= 10$
$\frac{10}{15} = \frac{2}{3}$
Answer: $\frac{2}{3}$

10 Finding the rate of interest

Isla invested £17,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £20,412.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $17500 \times m^2 = 20412.00$
$m^2 = 20412.00 \div 17500 = 1.1664$
$m = \sqrt{1.1664} = 1.08$
Rate = $(1.08 - 1) \times 100 = 8\%$
Answer: 8% per year