Day 158 **S Tier 10-a-day: Model Solutions**

1 Successive percentage change

The price of a laptop is decreased by 15%.
The new amount is then decreased by 10%.
Work out the overall percentage change.

Multipliers: 0.85 and 0.9
0.85 × 0.9 = 0.765
0.765 is a decrease of 23.5%
Answer: 23.5% decrease

2 Completing the square

(a) Write $4x^2 + 8x + 6$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 4x^2 + 8x + 6$

(a) Take out the factor 4 from the x terms: $4(x^2 + 2x) + 6$
$x^2 + 2x = (x + 1)^2 - 1$, so $4x^2 + 8x + 6 = 4[(x + 1)^2 - 1] + 6$
$= 4(x + 1)^2 + 2$
(b) The minimum is when $x + 1 = 0$, so $x = -1$ and $y = 2$
Answer: (a) $4(x + 1)^2 + 2$ (b) $(-1, 2)$

3 Finding the rate of interest

Liam invested £8,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £8,323.20.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $8000 \times m^2 = 8323.20$
$m^2 = 8323.20 \div 8000 = 1.0404$
$m = \sqrt{1.0404} = 1.02$
Rate = $(1.02 - 1) \times 100 = 2\%$
Answer: 2% per year

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 20: circumference = $2 \times \pi \times 20 = 40\pi$
The two straight sides: $2 \times 120 = 240$
Boundary = $(240 + 40\pi)$ m
(b) The rectangle is 120 m by $2 \times 20 = 40$ m, so its area is $120 \times 40 = 4800$. Circle: $\pi \times 20^2 = 400\pi$
Area = $(4800 + 400\pi)$ m^2
Answer: (a) $(240 + 40\pi)$ m (b) $(4800 + 400\pi)$ m^2

5 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{1}{4}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 4 mornings.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{1}{4})^4 = \frac{3}{4}^4 = 0.31640625$
P(at least once) = 1 − P(never) = 1 − 0.31640625
= 0.684
Answer: 0.684

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6 Simple and compound interest

Chen invests £2,500 for 4 years at 2.5% per year simple interest.
Gareth invests £2,500 for 4 years at 2.5% per year compound interest.
Work out how much more interest Gareth earns than Chen.
Give your answer to the nearest penny. (Calculator)

Simple interest: £2,500 × 2.5% × 4 = £250

Compound: £2,500 × 1.025^4 = £2,759.53

Compound interest = £2,759.53 − £2,500 = £259.53

Difference = £259.53 − £250 = £9.53

Answer: £9.53

7 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 12x + 35}{x^2 - 3x} \times \frac{x^2 + 2x}{x - 5}$

Factorise: $\frac{(x - 5)(x - 7)}{x(x - 3)} \times \frac{x(x + 2)}{(x - 5)}$

Cancel $(x - 5)$ and x

$= \frac{(x - 7)(x + 2)}{(x - 3)}$

Answer: $\frac{\mathbf{(x - 7)(x + 2)}}{\mathbf{(x - 3)}}$

8 Area and quadratic equations

The diagram shows a rectangle of length $(x + 1)$ cm, width $(2x - 3)$ cm and area 12 cm^2.
(a) Show that $2x^2 - x - 15 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(x + 1)(2x - 3) = 12$

Expand: $2x^2 - x - 3 = 12$, so $2x^2 - x - 15 = 0$

(b) Factorise: $(x - 3)(2x + 5) = 0$

$x = 3$ ($x = -\frac{5}{2}$ is not possible as a length would be negative)

Length = 4 cm and width = 3 cm

Diagonal = $\sqrt{4^2 + 3^2} = \sqrt{25}$ = 5.00 cm (3 s.f.)

Answer: (b) 5.00 cm

9 Surds and fractions

Simplify $\frac{1}{2 + \sqrt{3}} - \frac{2}{2 - \sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Numerator: $(2 - \sqrt{3}) - 2(2 + \sqrt{3})$

$= -2 - 3\sqrt{3}$

$\frac{-2 - 3\sqrt{3}}{1} = -2 + 3\sqrt{3}$

Answer: $\mathbf{-2 + 3\sqrt{3}}$

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10 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 43°.
B is a point on the same line, 40 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 24°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 43° = \frac{h}{x}$, so $x = h \div \tan 43°$

From B: $\tan 24° = \frac{h}{x + 40}$, so $x + 40 = h \div \tan 24°$

Subtract: $40 = h \div \tan 24° - h \div \tan 43° = h(2.2460 - 1.0724)$
$h = 40 \div 1.1737 = 34.1$ m (1 d.p.)
Answer: 34.1 m