Day 157 **S Tier 10-a-day: Model Solutions**

1 Percentage change

The price of a phone is decreased by 5%.
The new price is then decreased by 60%.
Work out the overall percentage change in the price.

Multipliers: 0.95 and 0.4
Overall multiplier = 0.95 × 0.4 = 0.38
0.38 − 1 = −0.62, so an overall decrease of 62%
Answer: 62% decrease

2 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 25 is taken. Work out how many come from Year 8.
(b) A different stratified sample has 24 from Year 7. Work out the size of this sample.

Total = 400 + 250 + 250 + 350 = 1250

(a) Year 8: $\frac{250}{1250} \times 25 = 5$

(b) Fraction sampled = 24 ÷ 400 = $\frac{3}{50}$, so sample size = $\frac{3}{50} \times 1250 = 75$

Answer: (a) 5 (b) 75

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 26°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 26°
Angle AOB = 180 − 2 × 26 = 128°
(b) The angle at the circumference is half the angle at the centre: 128 ÷ 2 = 64°
Answer: (a) 128° (b) 64°

4 Composite functions

f(x) = 2x − 4 and g(x) = x^2 − 8
(a) Find gf(x). Give your answer in the form $ax^2 + bx + c$.
(b) Solve gf(x) = 8

(a) gf(x) = g(2x − 4) = $(2x - 4)^2 - 8$
= $4x^2 - 16x + 8$
(b) $(2x - 4)^2 = 16$, so 2x − 4 = ±4
2x − 4 = 4 or 2x − 4 = −4
x = 0 or x = 4
Answer: (a) $4x^2 - 16x + 8$ (b) x = 0 or x = 4

5 Pressure

A concrete slab in the shape of a cuboid weighs 570 N.
Its smallest face has area 150 cm^2. Its largest face has area 2,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 2,500 cm^2 = 0.25 m^2
(a) Greatest pressure is on the smallest face: 570 ÷ 0.015 = 38,000 N/m^2
(b) Least pressure is on the largest face: 570 ÷ 0.25 = 2,280 N/m^2
Answer: (a) 38,000 N/m^2 (b) 2,280 N/m^2

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6 Where a line meets the axes

The line L has equation $8x - 15y = 120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $8x = 120$, so $x = 15$. A is $(15, 0)$
At B, $x = 0$: $-15y = 120$, so $y = -8$. B is $(0, -8)$
(b) $AB^2 = 15^2 + 8^2 = 289$
$AB = \sqrt{289} = 17$
Answer: (a) A(15, 0), B(0, −8) (b) 17

7 Reverse percentages

The price of a coat is decreased by 40%.
The new price is then increased by 12%.
The final price is £940.80.
Work out the original price of the coat.

Multipliers: 0.6, 1.12
Overall multiplier = 0.6 × 1.12 = 0.672
Original = final ÷ multiplier = £940.80 ÷ 0.672
= £1,400
Answer: £1,400

8 Transformations of graphs

The graph of $y = f(x)$ has a minimum point at (4, 1).
Write down the coordinates of the turning point of the graph of
(a) $y = f(x + 2)$
(b) $y = f(-x) + 2$

(a) Start with the turning point (4, 1)
Apply the transformation to the point: (2, 1)
(b) $f(-x)$ changes the x-coordinate to -4, and $f(x) + 2$ changes the y-coordinate to 3
Turning point: $(-4, 3)$
Answer: (a) (2, 1) (b) (−4, 3)

9 Probability and quadratic equations

A bag contains some counters. 7 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{7}{40}$.

Work out how many counters are white.

Let there be n counters altogether
$P(\text{both green}) = \frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)}$
$\frac{42}{n(n-1)} = \frac{7}{40}$, so $n(n-1) = 240$, giving $n^2 - n - 240 = 0$
$(n - 16)(n + 15) = 0$, so $n = 16$ (n cannot be negative)
Number of counters that are white = $16 - 7 = 9$
Answer: 9

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10 Trigonometric graphs

The diagram shows the graph of y = a sin bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 130°.

(a) The maximum value is 4, so a = 4
The graph completes 3 full cycles between 0° and 360°, so b = 3
(b) y = 4 sin (3 × 130°) = 4 sin 390°

sin 30° = $\frac{1}{2}$, so y = 2

Answer: (a) a = 4, b = 3 (b) y = 2