Day 156 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 15:45 (London time).
It takes 5 hours 30 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 15:45 + 5 h 30 min = 21:15
Sydney is 9 hours ahead of London, so add 9 hours
06:15 the next day
Answer: 06:15 the next day

2 Area of a segment

The circle has centre O and radius 9 cm. A and B are points on the circle and angle AOB = 60°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{60}{360} \times \pi \times 9^2 = 42.412$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 9 \times 9 \times \sin 60° = 35.074$ cm^2

Segment = sector − triangle = 42.412 − 35.074 = 7.3375
= 7.34 cm^2 (3 s.f.)
Answer: 7.34 cm^2

3 Quadratic graphs

The curve $y = x^2 + 2x - 3$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0: $x^2 + 2x - 3 = 0$, so $(x + 3)(x - 1) = 0$
$x + 3 = 0$ or $x - 1 = 0$, so $x = -3$ or $x = 1$
(b) The turning point is halfway between A and B: $x = (-3 + 1) \div 2 = -1$
When $x = -1$: $y = (-1)^2 + 2 \times (-1) - 3 = -4$
Answer: (a) A(−3, 0), B(1, 0) (b) (−1, −4)

4 Averages with algebra

Here are five numbers: x, 2x − 2, x + 1, x + 6, 2x + 2.
The mean of the five numbers is 14.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (x) + (2x − 2) + (x + 1) + (x + 6) + (2x + 2) = 14 × 5 = 70
7x + 7 = 70, so 7x = 63 and x = 9
The numbers are 9, 16, 10, 15, 20
(b) In order: 9, 10, 15, 16, 20, so the median is 15
Answer: (a) x = 9 (b) 15

5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 5^2 + 20^2 = 425$
$AG^2 = AC^2 + CG^2 = 425 + 4^2 = 441$
AG = 21 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{425}}$

angle GAC = 11.0° (1 d.p.)
Answer: (a) 21 cm (b) 11.0°

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6 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 40%.
Dina earns £76,500 in a year.
Work out how much income tax Dina pays.

Taxed at 20%: £30,000
20% of £30,000 = £6,000
Taxed at 40%: £76,500 − £12,500 − £30,000 = £34,000
40% of £34,000 = £13,600
Total tax = £6,000 + £13,600 = £19,600
Answer: £19,600

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -1$ and $x = 6$, so $y = a(x + 1)(x - 6)$
When $x = 0$, $y = -6$: $a \times 1 \times (-6) = -6$, so $a = 1$
$y = (x + 1)(x - 6)$
$y = x^2 - 5x - 6$
Answer: $y = x^2 - 5x - 6$

8 Radius from chord and height of arc

A circular arc has a chord AB of length 14 cm.
The greatest distance between the chord and the arc is 1 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 1
AM = 14 ÷ 2 = 7. Pythagoras in triangle OMA: $r^2 = (r - 1)^2 + 7^2$
$r^2 = r^2 - 2r + 1 + 49$
2r = 50, so r = 25 cm
Answer: 25 cm

9 Combining means

The mean of the marks in a test for the boys is 51.
The mean of the marks in a test for the girls is 40.
The mean for all of the boys and girls together is 45.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = 51p + 40q, and the overall mean gives 51p + 40q = 45(p + q)
6p = 5q
p : q = 5 : 6
Answer: 5 : 6

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10 Tangent to a circle: proof

Show that the line $y = x + 12$ is a tangent to the circle $x^2 + y^2 = 72$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x + 12$ into $x^2 + y^2 = 72$: $x^2 + (x + 12)^2 = 72$

Expand: $2x^2 + 24x + 72 = 0$

$b^2 - 4ac = 24^2 - 4 \times 2 \times 72 = 576 - 576 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-24}{2 \times 2} = -6$, and $y = (-6) + 12 = 6$. The point is $(-6, 6)$

Answer: $(-6, 6)$